Effective Strategies for Teaching Mathematics Content to English Language Learners: Use Problem Solving to Develop Conceptual Understanding

Presented at the

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Compiled and Presented by

Susie W. Håkansson, Ph.D.
California Mathematics Project
University of California, Los Angeles
TODOS: Mathematics for All

Model 1

$1 + 3 + 5 + \cdots + (2n-1) = n^2$
Proof by Mathematical Induction

Prove that: $1 + 3 + 5 + \cdots + (2n-1) = n^2$.

To prove the above statement by mathematical induction, we need to show two things:

1. The formula holds for $n = 1$.
2. If the formula holds for n, then the formula holds for $n+1$.

To show 1: If $n = 1$, then substitute $n = 1$ in the above statement and we see that $1 = 1^2$.

To show 2:

Assume that $1 + 3 + 5 + \cdots + (2n-1) = n^2$.

We need to prove that $1 + 3 + 5 + \cdots + (2n-1) + (2[n+1]-1) = (n+1)^2$.

$$\mathbf{1 + 3 + 5 + \cdots + (2n-1)} + (2[n+1]-1) = \mathbf{n^2} + (2[n+1]-1) \quad \text{(By substitution)}$$

$$= n^2 + (2n+2-1)$$

$$= n^2 + 2n + 1$$

$$= (n+1)^2$$

We have thus shown that the formula holds for $n+1$.

Therefore, by mathematical induction $1 + 3 + 5 + \cdots + (2n-1) = n^2$.

Model 2

$1 + 2 + 3 + \cdots + n = n(n+1)/2$

Model 3

How many rectangles are there in a 5 x 5 grid? What about an n x n grid?

1. Need to build on the formula that the sum of the first n natural numbers is $n(n+1)/2$, or $1 + 2 + \cdot \cdot \cdot + n = n(n+1)/2$.

2. Take a 1 x 1 square grid. How many rectangles are there embedded in the figure?

3. Take a 2 x 2 square grid. How many different rectangles are there embedded in the figure? A 2x1 is different than a 1x2.

4. Take a 3 x 3 square grid. How many different rectangles are there in a 3 x 3 square grid? How do you know you have all of them? How did you ensure that all rectangles are accounted for?

5. What patterns are emerging? Go on to a 4 x 4 square grid.

6. What may emerge is that the number of rectangles equals $(1+2+\cdots+4)^2$.

7. What do you predict for the number of rectangles in a 5 x 5 grid?

8. What about an n x n grid?

9. How many of you think that $(1+2+3+4+5)^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2$? How do you know whether or not that's true?

10. What about $(3+4+5)^2 = 3^2 + 4^2 + 5^2$?

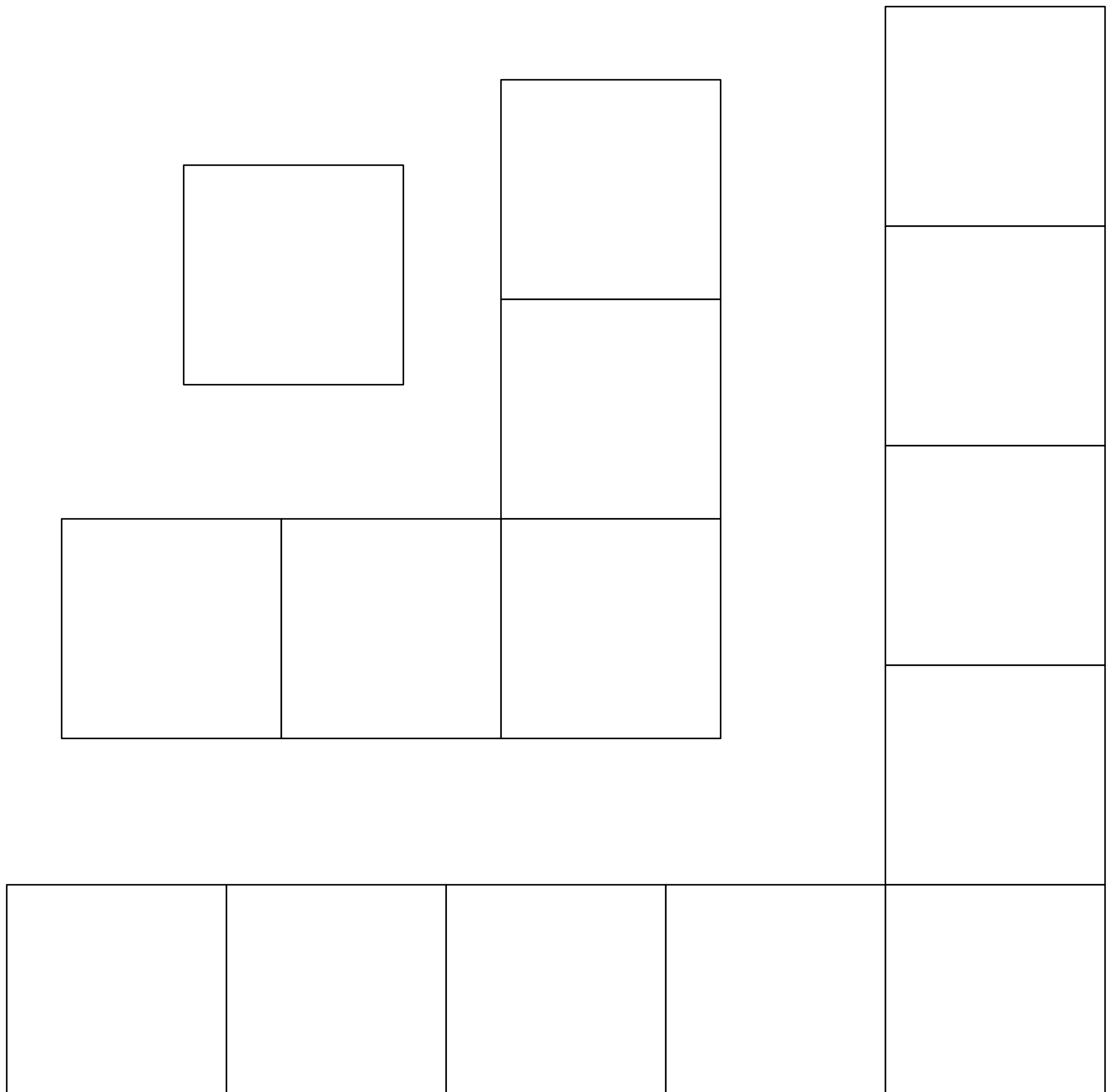
11. How many of you think that $(1+2+3+4+5)^2 = 1^3 + 2^3 + 3^3 + 4^3 + 5^3$? How do you know whether or not that's true? What about $(3+4+5)^2 = 3^3 + 4^3 + 5^3$?

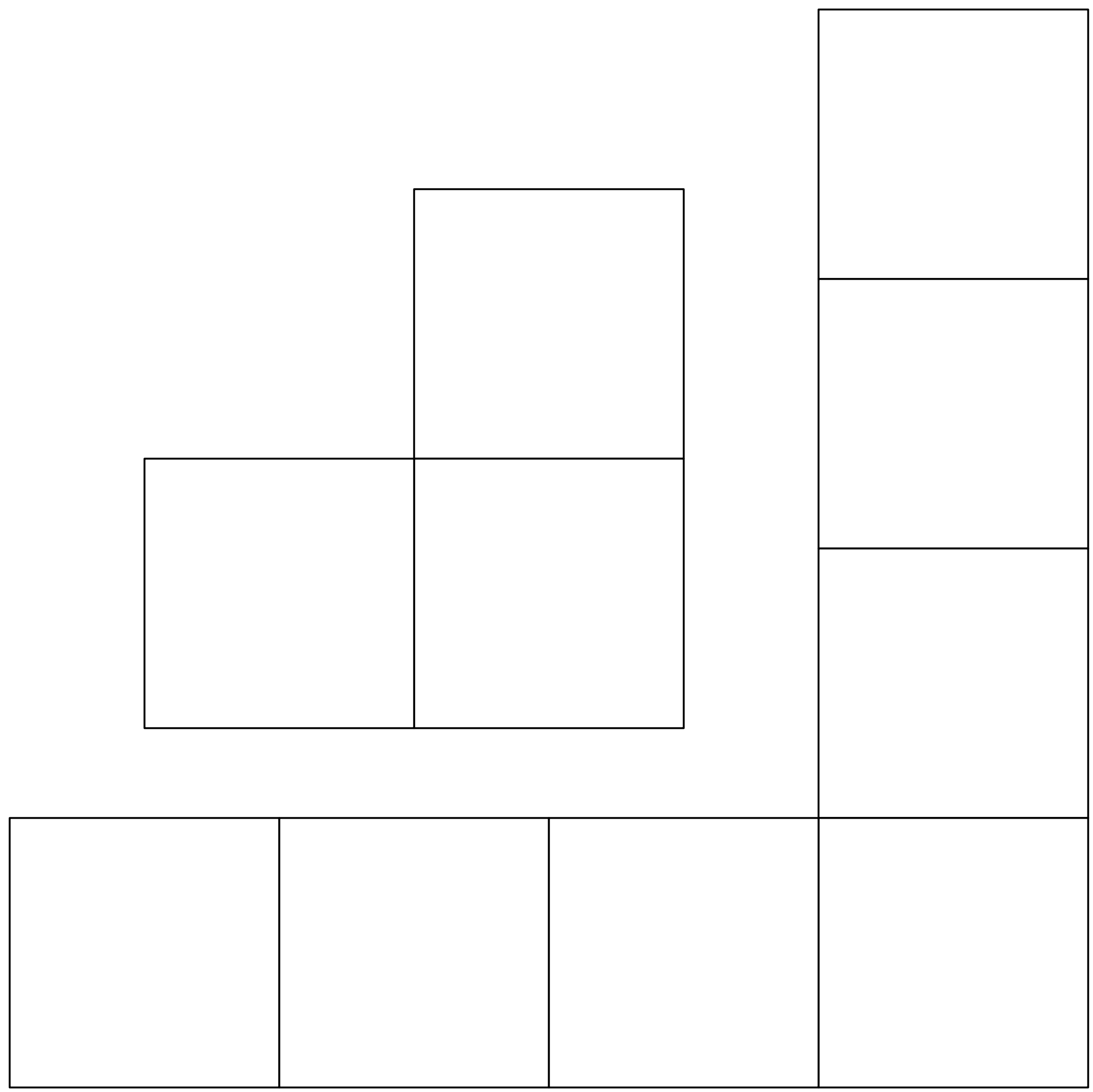
12. Go back to the grids and look at a specific method to count the number of rectangles (number of rectangles whose bottom right vertex is at a specific point).

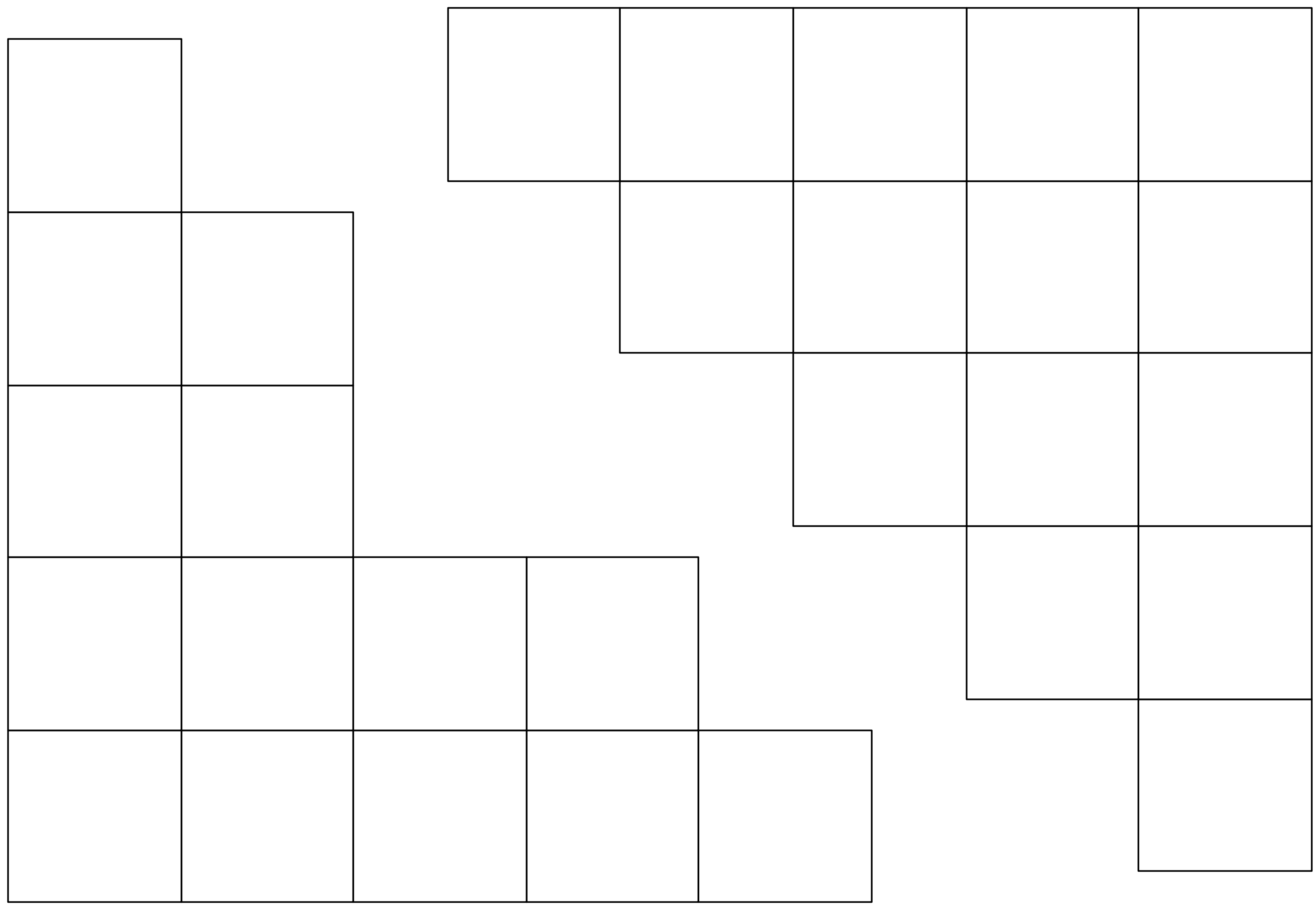
13. Looking at the numbers generated, determine a systematic way to show that the sum of the rectangles embedded in an n x n grid is $(1+2+3+\cdots+n)^2$.

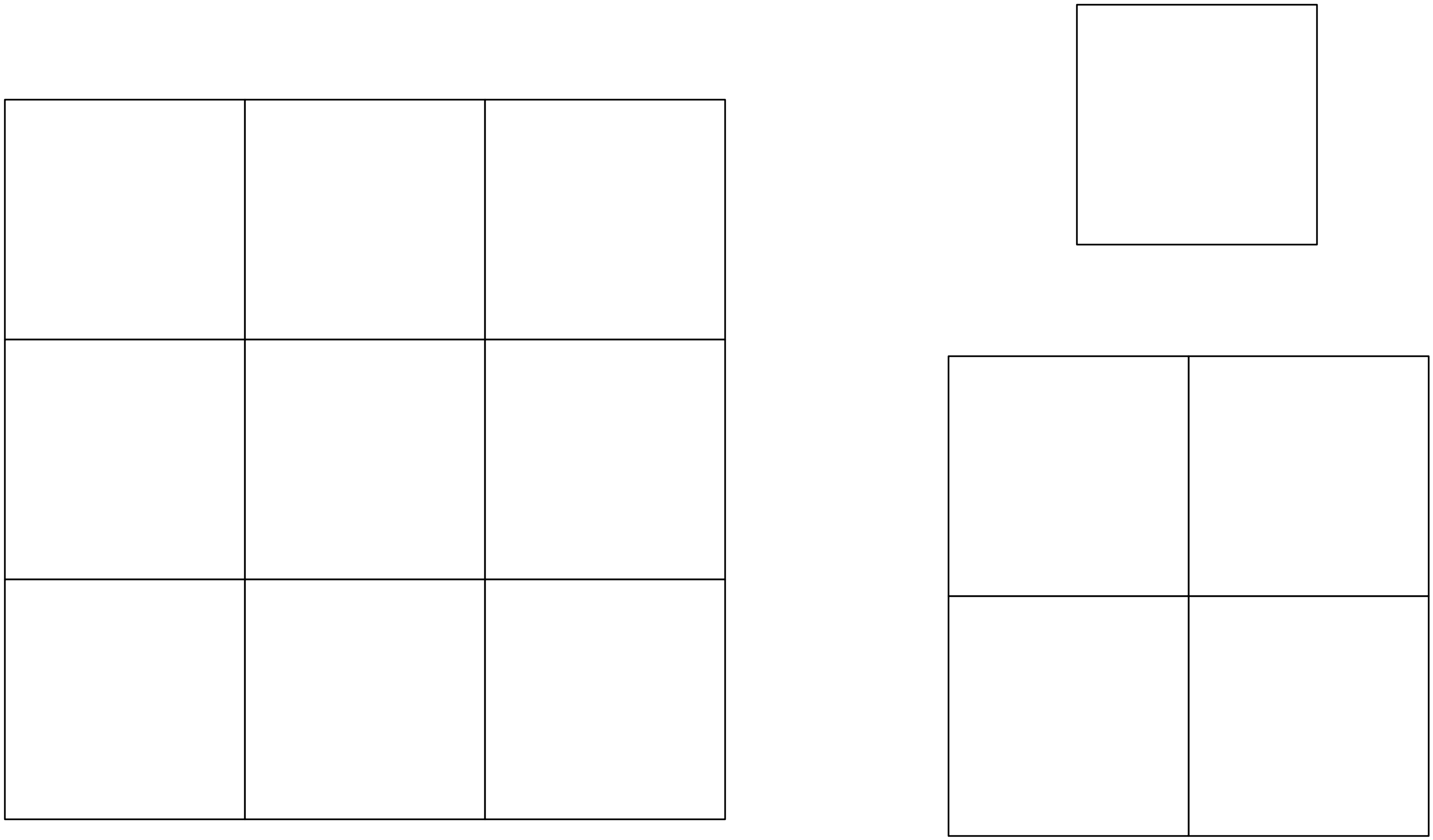
14. Looking at the numbers generated, determine a systematic way to show that the sum of the rectangles embedded in an n x n grid is $1^3 + 2^3 + 3^3 + \cdots + n^3$.

15. Use mathematical induction to prove $(1+2+3+\cdots+n)^2 = 1^3 + 2^3 + 3^3 + \cdots + n^3$.









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Description

Understand the language challenges that Academic English presents to English Language Learners. Use specific strategies to make the content comprehensible while developing understanding and learning of a mathematics formula. Understanding supports learning when teachers and students can derive the formula.

We are usually more easily convinced by reasons we have found ourselves than by those which have occurred to others.

Blaise Pascal

An Approach Supportive of English Learners

Teachers learn to amplify and enrich--rather than simplify--the language of the classroom, giving students more opportunities to learn the concepts involved.

Aída Walqui, Teacher Quality Initiative

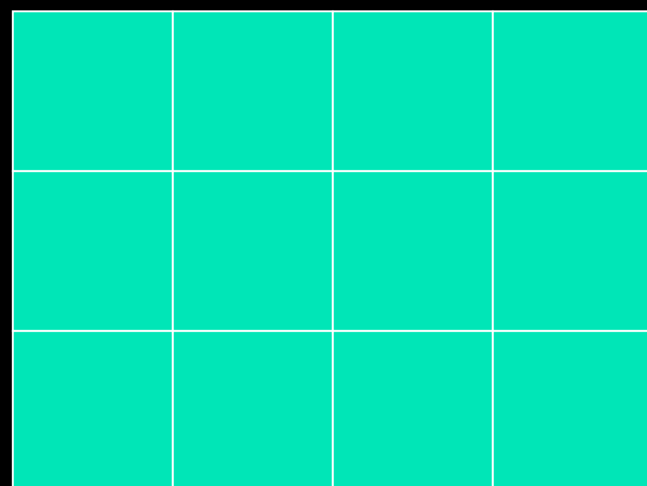
Mathematics Language

- Problem solving
- Conceptual understanding
- Visual model
- Example
- Mathematical induction
- Consecutive odd integers
- T-table

Mathematics Language

- Term
- Generalization
- Conjecture
- Proof without words/formal proof
- Triangular numbers
- Consecutive natural numbers
- Square grid
- Embedded

Visual Model



3 x 4 = 12

Students must develop the following characteristics:

- Interest in finding solutions to problems
- Confidence to try various strategies
- Willingness to risk being wrong at times
- Ability to accept frustrations that come from not knowing

Students must develop the following characteristics:

- Willingness to persevere when solutions are not immediate
- Understanding of the difference between not knowing the answer and not having found it yet.

About Teaching Mathematics, Marilyn Burns

The Challenge for English Learners

- Conversational fluency
- Discrete language skills
- Academic language proficiency

Supporting ESL Students in Learning the Language of Mathematics,

Jim Cummins

Mathematics Language vs. English

- Is zero a number?
  - I own a number of algebra books.
  - I have a number of friends.
- Is a straight line a curve?
  - English: a straight line isn't a curve
  - Mathematics: the simplest example of a curve

Mathematics Language vs. English

- What is a line?
  - English: any line segment
  - Mathematics: line is an infinite line
- Multiplying . . .
  - English: repeated addition--bigger
  - Mathematics: bigger, smaller, or neither

Mathematics Language vs. English

- Dividing . . .
  - English: cut into pieces
  - Mathematics: same as multiplication (dividing by a non-zero number is multiplying by its reciprocal).
- "Ameobas multiply by dividing"

Mathematics Language vs. English

- “a”
  - Problem: Show that a number divisible by 6 is even.
  - Answer: 42
  - Why is that not a proof?
  - “a” means “every”
  - The interpretation “some” is too trivial.

Mathematics Language vs. English

- "or"
  - Coffee or tea?
  - Are you coming or going?
  - Was that your father or father-in-law?
  - Do it now or later.

Mathematics Language vs. English

- "or"
  - English: "or" is exclusive.
  - Mathematics: by convention "or" is inclusive ("A or B" is true if A or B or both is the case).

Teaching the Language of Mathematics

Two-way and reciprocal: Mathematical knowledge is developed through language, and language abilities can and should be developed through mathematics instruction.

Teaching the Language of Mathematics

- Effective academic language instruction for ELL students is build on three fundamental pillars:
  - Activate prior knowledge/build background knowledge
  - Access content
  - Extend language

Prior Knowledge

- Foundation of learning
- Impact on English language learners
- Strategies to activate prior knowledge and building background knowledge

Access Content

How can teachers make the powerful and precise language of mathematics comprehensible for students who are still in the process of learning English?

Access Content

- Scaffold students' learning by modifying the input
  - Demonstration/modeling
  - Hands-on manipulatives, technology
  - Whole class; small groups
  - Visuals
  - Language clarification
  - Dramatization/acting out

Extend Language

- Creating mathematical language banks
  - Focus on meaning
  - Focus on form
  - Focus on use
- Taking ownership of mathematical language by means of “reporting back.”
- Mastering the language of mathematical assessment.

Quick Write/Discuss

How can we assist teachers to increase their effective strategies for teaching mathematics content to English language learners?