

## TABLE OF CONTENTS

### SECTION 1 – BACKGROUND

Page 2

Understanding the problems that English Language Learners sometimes encounter when learning mathematics in their second language.

### SECTION 2 – PURPOSE AND STRUCTURE OF TEACHER’S GUIDE

Page 6

The purpose and goals of this teacher’s guide, including a conceptual framework for the guide.

### SECTION 3 – GENERAL TEACHING STRATEGIES

Page 8

Strategies that include research-based “what works” teacher actions for English Language Learners as well as for all students who often struggle when learning mathematics concepts. Strategies focus on the learning of mathematics by Hispanic students.

### SECTION 4 – TEACHING MATHEMATICS CONCEPTS ON THE EXIT-LEVEL TAKS TEST

Page 16

Detailed strategies for teaching mathematics concepts that are tested on the Grade 11 Exit Level TAKS Mathematics Test for English Language Learners. Using each of the 10 mathematics concept objectives from the exit test as focus items, this section includes necessary mathematics vocabulary, specific teaching strategies, examples of performance tasks and projects, and appropriate assessment methods for English Language Learners. Links to other valuable statewide resources are included.

# **DRAFT**

## SECTION 1 -- BACKGROUND

It is recognized that some Spanish-speaking groups prefer to be called Spanish, Hispanic, Latino, Hispanic Americans, or possibly by another name referring to their country of origin (e.g., Cuban or Columbian). Since the U.S. Census Bureau uses the term Hispanic to refer to all Spanish-speaking groups, regardless of race, the term Hispanic will be used in this teacher's guide. The Hispanic population is the fastest and largest growing minority in the United States. According to the 2000 census, approximately 40 million Hispanics live in the U.S. with more than 40% of Hispanic families reporting to have children under 18 years old. Some Hispanic students live at or below the poverty line, which may lead to issues connected to low socioeconomic status. English language learners (ELLs) among Hispanic students may face challenges related to their learning of mathematics. Many school-related problems that the English language learner (ELL) experiences may be related to cultural differences, difficulties in learning the English language, or minimal prior knowledge of mathematics concepts. This section will help teachers better understand some of the concerns about teaching mathematics to ELLs. Even though the following information specifically targets Hispanic ELLs, these ideas may be applied to many students having difficulties learning mathematics.

### Variables that impact English language learners

There are many variables that impact the ELL as a student, other than ethnicity. Some of these include the socioeconomic class, geographic region, primary language, religion, family structure, and number of generations in the United States. Many Hispanic children come to school with less developed scholastic skills and little or no early childhood education. In addition, some Hispanic parents exhibit low educational expectations for their children and do not emphasize academic achievement.

- Family connections are important in the Hispanic culture

Hispanic youth feel a moral responsibility and commitment to their families. They are united by customs, language, religion and values and are likely to adopt their parents' commitment to religious and political beliefs, occupational preferences, and lifestyle. Family commitment involves loyalty, a strong support system, respect of elders, and an obligation to care for family members. Stereotyped sex roles still exist among many Hispanics where the male is perceived as being dominant and the female as nurturing. Cultivating personal relationships and alliances is also important to Hispanics. They would rather submit to peer pressure and choose not to do well in school than face humiliation by being called a "whitie" or "school-boy." Values like bonding with family and friends are in conflict with that of mainstream adolescents who strive for independence. Therefore, identity formation and individuality are challenging and sometimes problematic for the Hispanic student. The family's economic situation or peer rejection may also cause Hispanic youth to have a low self-image. Lack of self-

# DRAFT

confidence may cause ELLs to have lower educational expectations for themselves, and therefore they may not reach their academic potential.

- Family obligations

Many Hispanic youth will say that school is important to them, but sometimes family and economic issues become a priority. For example, the need for financial support may entice youth at age 13 to work in the family yard business rather than go to school. Some youth work long hours at night performing low-skilled jobs and are still expected to go to school during the day, trying to concentrate on classroom instruction. Often there are extended families living in the same household. Older children are expected to take care of (baby-sit) younger children and do household chores. In addition, at home there is seldom any privacy or time to do homework.

- Access to technology

Hispanic households have limited access to updated technology and educational aids. About 40% of Hispanic households have computers, compared to more than 70% of Caucasian households. The television is usually tuned to the Spanish channel with little or no access to educational channels. In addition, since Hispanic families may be of low socioeconomic status, other educational resources such as books, software programs, mathematics games, etc. may not be readily available to use for enrichment or extra help.

## Impact of language on mathematics learning

It is well known that it usually takes an ELL more than *one year* to develop conversational language and *five to seven years* to develop sufficient academic language to learn in English. The ELL may have problems with mathematics language because it uses technical terms including homonyms and synonyms. The English language structures such as word order and syntax are sometimes different than the student's native language. In addition, the teacher may be using idioms, figurative language, and regional dialects that can confuse the ELL. When translating words literally, without regard to language context, the semantics is sometimes lost. Examples include: many different English words that imply "add," (e.g., *plus, combine, and, sum, and increase by*); words with multiple meanings like "fix" or "table:" and logical connectors (e.g., *therefore, consequently, if, because, and however*) used in mathematical problems. Lack of English language knowledge may also result in low self-confidence. Consequently, ELLs sometimes have reservations about participating and interacting in class, asking questions, attempting a task, showing work, and explaining answers, due to limited vocabulary and language proficiency.

# DRAFT

- More time is required to interpret new concepts and vocabulary

The ELL needs more time to decipher and understand the language involved with a mathematics concept or word problem. Sometimes mathematics terms, phrases or abstract ideas have no direct translation to the student's native language. Therefore, it is difficult for the student to stay at the same pace in the classroom as a native English speaker. Without a rich mathematics vocabulary, the ELL will need more time to keep up with native English speakers. Every new term they learn must be embedded in familiar contexts, and this takes time when working in a second language.

- Grade placement and academic background knowledge

When ELLs enter the U.S., they are usually placed at a grade level appropriate to their age, not according to their academic background. If the background knowledge needed to be successful at that grade level is not sufficient, the result can be frustrating for the student. For example, as a 14-year old, they may be placed in eighth grade, even though their mathematics knowledge is at the sixth-grade level and their English reading skills are at the fourth-grade level. Remediation of language and prior content knowledge requirements may be necessary for the student to be academically successful.

## Mathematics content related issues

English language learners may bring in mathematic concepts and skills that may conflict with those used in the U.S. classroom. In mathematics, inconsistencies may occur in use of symbols, algorithms, measurement systems and sequencing of content.

- Use of symbols and numbers

The formation of numbers varies from country to country. An example of difference in numbers includes the numbers one and seven where one is a straight vertical line with a base and an acute angled line to the left of the vertical line. This may in some instances be confused with the way the seven is formed. To distinguish between the two numbers, a line is placed through the middle of the vertical line. See figures below:



In some cases, the placement of decimal points and commas differ in other countries. For example, the number 1.000,50 in some countries would translate to 1,000.50 in the United States.

# ***DRAFT***

- Use of algorithms

The subtraction and division mathematical operations are formatted and taught differently in other countries. In some countries, subtraction borrows from the bottom numbers rather than the top numbers like in the United States. Also, in division, the dividend and divisor are sometimes reversed and the answer may be placed below the dividend.

- Measurement system

All Hispanic countries use the metric system and do not teach the English measuring system in schools, and so ELLs often have no prior experience in working with the U.S. measurement system. In addition, currency (peso versus dollar), temperature (Celsius versus Fahrenheit), and time (military-style 24-hour clock versus the U. S. 12-hour clock with AM and PM) are not universal systems among countries.

- Variation in instructional strategies and topics

Mathematics curriculum in other countries is sequenced in a different manner. Calculation may be emphasized more than concept understanding, so they may be reluctant to show work. Mathematics curriculum is not spiral in many countries, and students may understand number operations, but geometry concepts may be new ideas to some students. Many ELLs have not seen or worked with manipulatives, and they may not take the lesson seriously. Estimating skills may not have been previously emphasized.

# ***DRAFT***

## SECTION 2 – PURPOSE AND STRUCTURE OF TEACHER’S GUIDE

This teacher’s guide is designed to help teachers make mathematics more meaningful and understandable for the English language learner (ELL). The initial focus of this guide is for grades 7-11, since these are the grade levels where the mathematics content is taught or reviewed for the Mathematics Grade 11 Exit Level Test. In addition to understanding the common issues that the ELL faces in a mathematics classroom (see Section 1 of this guide), we have included general teaching strategies (see Section 3 of this guide) that help the ELL when learning mathematics content. In Section 4, we provide a more detailed plan for teaching the ten mathematics objectives covered on the Exit TAKS Test.

A conceptual framework was created to focus on four main areas in this teacher’s guide and it includes:

Mathematics content for grades 7-11

- Focus on the mathematics concepts covered on the Exit Level TAKS

Understanding the English language learner in relation to learning mathematics

- Understand background and culture of ELLs
- Diagnose both language needs and current mathematical understanding

Instructional strategies for learning mathematics

- Develop appropriate mathematical skills and applications
- Develop and improve vocabulary
- Improve English language acquisition
- Support diverse learning styles and multiple intelligence

Multiple Assessments of Student Learning in Mathematics

- Use a variety of appropriate assessment tools
- Use out-of-class projects to connect the mathematical content to contextual experiences of student and family-life of the English Language Learner
- Ensure assessments are as free of bias as possible.

This teacher’s guide is part of a large project by the Texas State University System, in collaboration with the Texas Education Agency. The project website, [www.tsusmell.org](http://www.tsusmell.org), provides more information on the overall project. The Mathematics for English Language Learners (MELL) Classroom Practices Framework was used as a guide in the development of this teacher’s guide. This document is also available on the

# ***DRAFT***

[www.tsusmell.org](http://www.tsusmell.org) website.

The ideas and teaching strategies discussed in the teacher's guide are based on research in the field of teaching mathematics to ELLs, as well as from the experiences of teachers in classrooms with ELLs, and other educators. Sul Ross University educators conducted an exhaustive review in this area in 2005, and the results of this review are available to interested readers at the following website:

[www.tsusmell.org](http://www.tsusmell.org)

Teachers who want to read more about what research studies have revealed about the teaching of mathematics to ELLs can access the website provided.

In addition, Sul Ross University educators developed a Quick Start Professional Development Module, which will be available on-line for use by teachers and teachers of teachers in the summer of 2005. This module will include numerous links to applicable ELL documents, which were prepared as a part of this overall project or by external experts in the field. To find out more about the Quick Start Module, look on the following website:

[www.tsusmell.org](http://www.tsusmell.org)

# ***DRAFT***

## **SECTION 3 – GENERAL TEACHING STRATEGIES**

This section provides research-based teaching strategies that mathematics teachers can adapt to reduce some of the barriers often found by English language learners (ELLs). It is based on the MELL Classroom Practices Framework, which is available on the [www.tsusmell.org](http://www.tsusmell.org) website. Teachers will find that many of the instructional techniques will also help other students having difficulty learning mathematics (and perhaps other subjects as well). It is important to know the cultural and academic backgrounds of all students, as well as their mathematic background knowledge, which may influence the student's learning. This section describes strategies for getting to know the English language learner as an individual, focuses on classroom practices, and details some general teaching strategies giving examples for their use.

### **The English language learner**

There are various strategies for getting to know the English language learner as an individual, and these are described in the following sections.

- Gather assessment data

At the beginning of the school year, assess the needs of ELLs in your classroom. At the time of enrollment, school personnel such as the counselor, registrar, and English as a Second Language (ESL) teacher collect important student documentation, past academic assessment records, and language assessment information (including language(s) spoken in the home), which are maintained in the student's permanent records. Teachers should review these documents to get a better idea of the ELLs current levels of achievement and language understanding. After reviewing the documentation, it may be necessary to administer a diagnostic mathematics test or the teacher can make informal assessments based on in-class individual performance to evaluate the extent of English language and mathematics understanding, as well as the comfort level of each ELL. More information can also be obtained from other teachers having the same students and possibly the student's parents or other relatives. There may be older children or relatives at home who are more skilled in English and who can also be a resource to help the ELL.

- Value student's background

Value and respect the language, customs, and culture of the ELL. Be knowledgeable about Spanish customs, traditions, and holidays, and use these as context to make mathematics meaningful for Hispanic students. The teacher's cultural sensitivity and awareness will help the Hispanic ELL feel comfortable in the classroom.



# DRAFT

- Encourage family involvement

Hispanic family involvement is critical for the student's academic success. Teachers should respect parents and view them as capable partners in their child's education. Contact parents at the beginning of the school year. Be the first to contact parents; don't wait for them to contact you. Make the first contact positive and welcome the family to come to the school at any time. Invite parents to attend a "parent orientation" meeting and go over expectations for the year. If you don't speak Spanish, ask a Spanish-speaking student, friend, teacher's aide, or other Spanish-speaking community member to help you contact those parents who only speak Spanish. Some suggestions for involvement in the classroom include: tutors, helpers, and field trip volunteers. . For continued involvement, have students conduct real-world projects which would include their families, involving such issues as automobile insurance, sales tax on items, installment payments on furnishings, etc.

- Use appropriate Spanish phrases to communicate with students

A teacher of Hispanic students does not need to speak or understand the Spanish language. However, using everyday conversational Spanish as "icebreakers" will help the Hispanic learner feel more comfortable in the mathematics classroom. Hispanic students may address you as "teacher" in addition to Mr., Mrs., or Miss as a sign of respect, or may use "maestro or maestra" for highest respect. The following phrases with their Spanish translations and pronunciations might be useful.

English phrase	Spanish translation	Pronunciation
Good morning	Buenos días	Bu-eh-nos dee-as
Good afternoon	Buenas tardes	Bu-eh-nas tar-des
Hello	Hola	O-la
How are you?	¿Cómo está? (one student) ¿Cómo están? (class)	Có-mo es-tá Có-mo es-tán
How is it going?	¿Qué tal?	Kay tal
What's wrong? What's up?	¿Qué pasa?	Kay pa-sa
Your name, please.	¿Su nombre, por favor?	Su nom-bre, por fa-vor
Do you understand?	¿Comprende?	Com-pren-de
Is it clear?	¿Claro?	Cla-ro
Of course!	¡Claro que sí!	Cla-ro kay see
It is easy!	¡Es fácil!	Es fah-seal
Ten minutes left to finish	Falta diez minutos para terminar	Fahl-tah dee-es mee-nu-tos pah-rah tehr-mee-nahr
Hurry up!	Apúrese	Ah-pu-rey-say

# DRAFT

English phrase	Spanish translation	Pronunciation
Listen!	¡Escuche! (one student) ¡Escuchen! (many students)	Es-ku-cheh Es-ku-chen
Come here	Ven Aquí	Ben ah-key
Come here now!	¡Ven aquí, ahora!	Ben ah-key ah-oh-rah
If you make a mistake	¡Válgame dios! ¡Ay Chihuahua!	Bal'-ga-meh dee-os a-ee chee-wa-wa
Very good!	¡Muy bien!	Mu-ee bee-ehn
Fantastic!	¡Fantástico!	Fan-tas'-tee-co
Magnificent!	¡Magnífico!	Mag-nee'-fee-co
Thanks for your hard work	Le agradezco su buen trabajo	Lay ah-greh-des-co su bu-ehn trah-bah-ho
Attention, please	¡Atención, por favor!	Ah-ten-see-ohn', por favor
Like this	Así	A-see'
Any idea?	¿Tiene idea?	Tee-e-nay ee-dey-ah
Tell me	Dígame	Dee'-gah-meh
For example	Por ejemplo	Por eh-hem-pleo
Here is your <u>homework</u> or classwork	Aquí está su <u>tarea</u>	A-key' es-ta' su tah-rey-ah
How do you say <u>office</u> in Spanish?	¿Cómo se dice <u>office</u> en español?	Coh-moh say dee-say <u>office</u> en es-pah-nyol
Book	Libro	Lee-bro
Pencil	Lápiz	Lah-piece
Paper	Papel	Pah-pel
Chair	Silla	See-yah
Bathroom	Baño	Bah-nyo

- Involve others in bridging the language gap between ELLs and English-speaking students

Enlist the support of English-speaking students to help the ELL. This can be done via role reversal, where they can develop empathy for what the ELL must overcome when learning mathematics in a different language. An example might be for you to develop sample mathematics word problems in a different language (such as German or even a make believe language) and have the students work in groups to try to solve the problems. Using another Latin-based language (such as Portuguese) might be even more effective, since the ELL student would have an advantage over the English-speaking student.

## Classroom Practices

There are various classroom practices that have been shown to be effective with the English language learner.

# DRAFT

- Establish a positive classroom climate

Create an emotionally safe learning environment that helps students feel secure and willing to take risks. Help students set realistic and manageable goals based on the student's ability. Involve students as active participants. Make the normal classroom sequence of activities structured and predictable. Display student work, make word walls and colorful posters or pictures for classroom decorations. Add color-coded learning supports when appropriate. Focus on communication, not errors; do not allow other students to correct errors. Give students responsibility for their own learning. Include cooperative grouping activities and other opportunities to work with peers. The room arrangement may be in clusters rather than rows of desks when appropriate to the lesson. Make learning relevant to the students' experiences. Frequently use models, music for motivational purposes or introducing a lesson, gallery-walks (poster-walks), and concept maps for reinforcing mathematic concepts. For beginning or newly arrived ELLs, use conversational native language when possible. Collaborate with other teachers who have the same ELL student in their classes.

- Everyday instructional and teacher practices

Speak slowly and clearly, using a regular rate and tone of voice. When speaking, try to face the student(s) and not speak while your back is turned to the class. Model correct usage and word order, limiting slang words or phrases. Avoid idiomatic expressions such as "pulling your leg" or "back-off." Use sincere facial expressions, gestures, and body language when speaking. Use longer pauses between phrases. Use shorter sentences with simple syntax. Carefully use logical connectors such as *if*, *because*, *as a result*, *in comparison*, *however*, and *consequently*, and give examples to support understanding. Stress high frequency vocabulary (e.g., *function*, *operation*). Check for comprehension and repeat or restate explanations. Provide a longer wait time for students to answer and process information. Praise students on simple responses that they give and for accomplishments in the classroom.

- Supplement and adapt existing materials

Provide culturally rich learning materials. Have educational resources such as books, software programs, mathematics games, prior knowledge (remedial) learning aids, and manipulatives available in the classroom. Highlight essential information and avoid distracters leaving clear, concise content. Simplify the language of instruction, not the concept being taught. Provide and explain (written and oral) lists of instructions for completing assignments. Have students work on each step before moving to the next, scaffolding or building meaningful connections between what they are learning and their personal experience. Simplify the language by incorporating pictures, charts, timelines, diagrams, maps, examples, hands-on activities, and manipulatives. When reading textbooks, have the ELL focus initially on the visuals rather than on the text. Assign a supportive partner to talk to the ELL about the pictures and graphs, point to key terms, and read the captions aloud.

# **DRAFT**

- Assignment modifications

Provide chapter outlines or anticipation guides to students at the start of a new topic. Have students write chapter summaries prior to the assessment of their understanding of that chapter's concepts. For beginning ELLs, have students look at the visuals in the chapter, rather than the words, writing about what they see. Have students write in their native language using English words as they develop the skills. Provide ELLs with a list of key vocabulary words prior to the lesson being taught, or have them develop their own lists as new vocabulary is needed. Allow or provide ELLs bilingual dictionaries. Encourage students to use technology and other sources to reinforce developing knowledge. Divide assignments into smaller segments. Create projects for real events and real audiences.

- Assessment modifications

Advise ELLs of exactly what they are expected to learn. Make assessment an integral part of instruction. Assemble a study guide using semantic mapping, which will help build relationships for key vocabulary, important concepts and connections. Provide multiple types of evaluation including observation, journals, group tasks, oral responses, real-world projects, rubrics, projects, portfolios, and checklists that can record growth over time. Reformat the test as needed (e.g., use more space for recording responses). Develop the test in shorter sections. Allow more time to complete the test, or give shorter tests. Don't take off points for misspelled words. Design tests using pictures and manipulatives to assess conceptual understanding so that success on the test is not heavily dependent on reading comprehension. Allow students to work in their primary language and then translate their solutions into English when possible.

## Teaching Strategies

Various teaching strategies have been shown to be effective for the English language learner.

- Identification and use of learning styles

Support multiple learning styles that include visual (seeing), auditory (hearing), kinesthetic (moving), and tactile (hands-on) categories. Learning style preference may depend on the ELL's culture and customs, mainstream societal influences or expectations, and their previous learning experiences. For visual learners, use multiple representations such as, pictures, graphic organizers, concept maps, flow charts, graphs, diagrams, and charts to augment instruction of a mathematics concept. Many ELLs prefer these instructional aids to help them learn the words and concepts at the same time. Tactile and kinesthetic learners can benefit by using a variety of mathematics manipulatives to develop concept understanding. For example, Cuisenaire rods help students see the relationship of the part to the whole when learning fraction concepts. And using a metric trundle wheel to measure the length of

## **DRAFT**

the hall in meters helps reinforce measurement concepts with a hands-on activity, rather than just solving a problem from a textbook. Allow ELLs to audiotape lessons or their own explanations about how they worked a problem. Auditory learners can review the taped material at a convenient time to reinforce their understanding.

- Grouping strategies

Many ELLs prefer to work with their classmates, rather than by themselves. Assigning (or having the student choose) a supportive learning partner is helpful. A partner can assist the ELL student with mathematics instructions, classroom procedures, as well as school procedures (e.g., schedules for bus, lunch, fire drills, etc). Partners can also aid English language development, enhance active participation in class, and welcome new students into the established learning community.

Cooperative learning is a strategy where students work together in a positive interdependent manner to accomplish a common goal. It differs from small group work in that each individual is expected to acquire a specific knowledge or skill and be able to demonstrate his or her accomplishment on their own beyond the end of the group task. Cooperative learning techniques particularly useful in teaching mathematics are think-pair-share, jigsaw, and structured problem solving. A list of additional strategies and explanations of their use are available on the website:

<http://www.utexas.edu/academic/cte/hewlettcls.html>

- Teach for conceptual understanding

Teachers should place emphasis on concept understanding using multiple representations, along with computational accuracy. Understanding why the mathematics procedure works is better than just knowing how to carry out the procedure to get the answer. For example, have students measure the diameter and the circumference of various containers with a circular base, and then ask them to find the ratio of the circumference to the diameter in each case. Discovering that this ratio is always  $\pi$  is more powerful for learning than the teacher giving students a formula and having them plug in numbers to calculate answers to mathematics problems. Some ELLs prefer discovering new mathematics concepts on their own, while others may learn better through a direct teaching approach. Timed drills or repetitive problems are rarely effective with ELLs and seldom help them retain mathematical knowledge for very long. Periodically during the lesson or at the end of the lesson, review important concepts and key vocabulary, and connect these to the objectives for the lesson.

- Enhance English language skills in the mathematics classroom

The ELL can learn mathematics concepts in English on a daily basis through content-based instruction (CBI). This will help them to develop both their linguistic ability and mathematics content. The basic ideas of Sheltered Instruction are often effective for ELLs. This model includes teacher preparation actions, strategies for classroom

# DRAFT

organization, and delivery of instruction, and is designed to make academic content understandable to ELLs. Emphasize quantitative relations, such as “more,” “less,” “three times as many,” “smaller,” etc. with models and manipulatives. These key words are important in mathematics and sometimes will help ELLs find solutions to mathematics problems. Avoid using negative statements, such as “all but,” “except,” and “which answer is not”, since these are confusing to the ELL. Also, when learning an algebraic algorithm, have copies of the words and steps for each student (or display words/phrases and steps on an overhead or computer) so that students can read and say the words aloud as they work through the steps. Supplement mathematics textbooks with English and Spanish books, as well as with videos and resources available on the Internet. Become familiar with second language acquisition theories and English as second language (ESL) teaching strategies.

- Use graphic organizers

Graphic organizers can be helpful for the ELL by showing relationships between mathematics concepts and vocabulary or by breaking down a larger concept into smaller parts. Examples include semantic webs, Venn diagrams, flowcharts, compare and contrast charts, T-charts, timelines, maps, and other diagrams.

- Directly teach math vocabulary

Teach mathematics vocabulary, not just conversational English. Introduce new vocabulary in a variety of ways such as making models, displays, diagrams, demonstrations, charts, and drawings of new words. Review previously-learned mathematics vocabulary when needed for the current lesson. Use brainstorming and define vocabulary used in a particular lesson in context. Have students restate definitions or meanings in their own words.

- Build vocabulary glossaries

Build glossaries using “meanings” of the words in everyday language or applications rather than direct definitions. The “meaning” should describe the word in such a way that anyone, who does not know what the word means, will understand. In addition a graphic organizer further reinforces vocabulary words. For example:

Glossary of Word Meanings and Descriptions			
English term	Spanish term	Description/meaning	Drawing/example

# DRAFT

- Create vocabulary games

Create a fun way to explore vocabulary using games. Types of games could include Jeopardy, Charades, Bingo, and Concentration. Other activities include pantomiming, acting out new words, or having students invent their own game, including sets of rules.

- Journal writing

Conclude class discussions with students writing entries in journals or an academic notebook. Journals in general need to be kept confidential (between the student and the teacher) so that students can also communicate important personal thoughts and feelings to the teacher. Journals can also provide insight into a student's mathematical thinking as part of an alternative assessment of student performance. Newcomers can write partial entries in their native language, but should use the journal as an opportunity to practice their writing in English. They might need assistance from their partner to write entries in English. Examples of topics might include the following:

- 1) What would you find in your home that would relate to today's topic?
- 2) Write the following math problem in words:  $x + 4 = 9$ .
- 3) Tell me the meaning of the word parallelogram in geometry, in every day English, and draw a picture.
- 4) Identify definitions and draw examples and non-examples of the vocabulary learned.
- 5) Create analogies using prior and new vocabulary (for example: square is to cube as rectangle is to prism).
- 6) List any prefix or suffix that appears regularly and write what it means.
- 7) List word origins (for example, linear comes from the word "line" and is Latin in origin from the word "linearis").

- Integrate other disciplines

Thematic units across the curriculum help support the ELLs understanding of mathematics and how it relates to other disciplines. Include literature with mathematics as themes. There is a wealth of materials available that covers mathematics topics using stories. An example would be the *Sir Cumference* series of books by Cindy Neuschwander. Teachers and students can read aloud and follow the words in a book, or copies of an article. Other examples include incorporating musical notes and timing when teaching fractions and using science concepts like density for teaching ratios.

# **DRAFT**

## **SECTION 4 - TEACHING MATHEMATICS CONCEPTS ON THE EXIT-LEVEL TAKS TEST**

This section contains the specific applications of the previous strategies (see Sections 1 and 3) to the mathematics content of the Grade 11 Exit Level TAKS Mathematics Test for English language learners (ELLs). Using each of the mathematics TAKS content objectives for the exit test as focus items, this section includes necessary mathematics vocabulary, specific teaching strategies, examples of performance tasks and projects, and appropriate assessment methods for ELLs. To access this information booklet on the Internet, [www.tea.state.tx.us/student.assessment/taks/booklets/index.html](http://www.tea.state.tx.us/student.assessment/taks/booklets/index.html) and click on the exit level test.

The Grade 11 Exit Level TAKS Mathematics Test information booklet contains ten TAKS objectives. These objectives state that the student will be able to:

1. describe functional relationships in a variety of ways;
2. demonstrate an understanding of the properties and attributes of functions;
3. demonstrate an understanding of linear functions;
4. formulate and use linear equations and inequalities;
5. demonstrate an understanding of quadratic and other nonlinear functions;
6. demonstrate an understanding of geometric relationships and spatial reasoning;
7. demonstrate an understanding of two- and three-dimensional representations of geometric relationships and shapes;
8. demonstrate an understanding of the concepts and uses of measurement and similarity;
9. demonstrate an understanding of percents, proportional relationships; probability, and statistics in application problems; and
10. demonstrate an understanding of the mathematical processes and tools used in problem solving.

The following information is provided for each of the above objectives:

Mathematics Content: A mathematical development of the topic and how it relates to previous learning and future learning in mathematics.

Mathematics Vocabulary: A list of appropriate English-Spanish mathematical terms and their meaning, and strategies for developing this vocabulary.

Teaching Strategies for Objective: A variety of appropriate teaching strategies for the mathematical content with an emphasis on good practices for English language learners and good strategies for teaching mathematics.

Assessment of Mathematics Objective: Examples of performance tasks and/or projects to connect the mathematics content to contextual experiences and family-life of the English language learner. Include appropriate examples of multiple-choice questions



# ***DRAFT***

similar to the Grade 11 Exit Level TAKS, including problems to solve, applications of the mathematics topic, problem solving situations, and student projects to integrate the mathematics to student and family-life. Pay particular attention to eliminating assessment bias. In addition, the Charles A. Dana Center developed assessment materials for algebra and geometry. This document is available at the following web site:

<http://www.tenet.edu/teks/math/clarifying/algebra1/alg1assess.pdf>

In addition to the strategies and ideas presented in this section, Texas State University at San Marcos educators developed a summary of professional development models used in the state of Texas. These models are actively used in school districts throughout the country and provide training programs that will help teachers of English language learners. Several of these professional development models include Sheltered Instruction (SIOP), Cognitively Guided Instruction (CGI), Family Math (EQUALS), and Everyday Math. Information on these programs is available at

[www.tsusmell.org](http://www.tsusmell.org)

Also, the National Council of Teachers of Mathematics (NCTM) published “Principles and Standards for School Mathematics” in 2000. This highly regarded document provides guidelines and examples of how students can learn important mathematical concepts and processes with understanding and is applicable for grades pre-kindergarten through Grade 12. Information on this document and on the NCTM philosophy for teaching mathematics can be found on the NCTM website at [www.nctm.org](http://www.nctm.org) and [www.illustrations.nctm.org](http://www.illustrations.nctm.org). These good teaching practices will also benefit English Language Learners.

**DRAFT**

## OBJECTIVE 1

## DESCRIBE FUNCTIONAL RELATIONSHIPS IN A VARIETY OF WAYS


Mathematics content:

## Prior mathematics knowledge requirements:

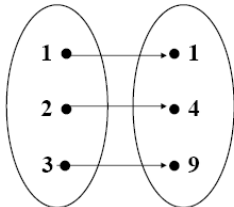

1. understand the concept of a variable;
2. write and solve equations with one and two variables, using concrete models and algebraic expressions;
3. write and solve inequalities with one and two variables;
4. represent points as ordered pairs;
5. graph points on an x,y-coordinate plane (complete graphs with axes, positive and negative values, and all four quadrants);
6. draw complete graphs of equations and inequalities on an x,y-coordinate plane;
7. record data for one and two variables in a table form;
8. understand functional relationships as a table of values;
9. identify patterns and proportional relationships between two variables; and
10. determine the nth terms of a sequence of numbers or geometric figures.

From pre-algebra activities in middle school, students should have a good understanding of variables, and how to set up and solve equations and inequalities. If students are weak in these areas, remediation may be required. Once students understand multiple ways to show a functional relationship between two variables, they will be prepared to learn Objectives 2-5 of this guide.

Minimum mathematics vocabulary needed for Objective 1:

<b>English term</b>	<b>Spanish term</b>	<b>Description/meaning</b>	<b>Drawing/example</b>
Cartesian coordinate system	sistema de Cartesiano coordenado	a rectangular system for graphing ordered pairs with x-coordinates and y-coordinates	
concrete models	representación física o algebraica	representations of functional relationships with manipulatives or real-world examples	algebra tiles, color tile manipulatives

**DRAFT**

English term	Spanish term	Description/meaning	Drawing/example
dependent variable	variable dependiente	the output variable whose value is determined by the independent variable (input variable) and the rule applied	income = (number of hours worked) times (\$ per hour) Dependent variable is income, which depends on the number of hours worked.
diagram	diagrama	a picture or drawing that represent the functional relationship between two variables	
equation	ecuación	a mathematical sentence that states that two expressions are equal	$y = -2x - 3$
function (functional relationship)	función	a relation where each input is paired with exactly one output and is based on some rule or description	$y = 2x - 3$ Each value for $x$ gives a different value for $y$
graph	gráfica	the solution set of an equation or inequality on a Cartesian coordinate system	
independent variable	variable independiente	the variable whose value is subject to choice (input variable)	income = (number of hours worked) times (\$ per hour) Independent variable is number of hours worked.

# DRAFT

English term	Spanish term	Description/meaning	Drawing/example								
inequality	desigualdad	a mathematical sentence that shows a greater than, greater than or equal, less than, or less than or equal relationship between two expressions	$y \geq 3x + 4$								
ordered pair	par ordenado	a pair of numbers in which the order they are written is important.	(3,6) is a different ordered pair than (6,3)								
relation	relación	a set of ordered pairs, which describe a relationship between two variables. For a relation, it is acceptable for each input value to be paired with more than one output	$A = \{(1,2), (2, 4), (3,6)\}$ Or $B = \{(1,2), (1,4), (1,8)\}$								
table	tabla	a display of mathematical data, usually in a vertical or horizontal manner	<table border="1" style="display: inline-table; vertical-align: middle;"> <tbody> <tr> <td>x</td> <td>2</td> <td>4</td> <td>6</td> </tr> <tr> <td>y</td> <td>5</td> <td>9</td> <td>13</td> </tr> </tbody> </table>	x	2	4	6	y	5	9	13
x	2	4	6								
y	5	9	13								
variable	variable	a symbol (usually a letter) used to represent an unknown number in a given set of numbers	in the expression, $4n + 2$ , the “n” is a variable								

### Strategies for learning this vocabulary:

1. write definitions in everyday language while still following correct mathematics;
2. use previously defined or common words in definitions and explanations;
3. have students develop self-made glossaries of new vocabulary in journals, picture cards, or charts;
4. as new vocabulary is introduced, add words and definitions with illustrations/explanations to classroom word wall;
5. repeatedly connect the words to mathematical symbols and examples;
6. tape record mathematical words, definitions and verbal examples, for students to play back when needed for extra support; and
7. examine words from Greek and Latin prefixes, roots, and suffixes.

# **DRAFT**

## Teaching strategies and examples for Objective 1

1. Use tables to describe relationships of one variable depending on another variable.

Example: A lawn care business charges \$25 to cut the grass and trim for a normal sized yard. Build a table to show that the amount of money earned (the dependent variable) depends on the number (#) of normal-sized lawns cut (the independent variable), as shown below: Students should fill in the values in the table.

<u># lawns cut</u>	<u>\$ earned</u>
1	25
2	50
3	75
4	100
.	.
10	?
.	.
$n$	?

Describe in words the connection that the amount of money earned depends on the number of lawns cut.

2. Gather and record data and determine functional relationships between quantities.

Example: have students work with a partner and measure the length of each other's foot, along with the height of each student. Consolidate data for the entire class, plot the data for each student on an  $x,y$ -coordinate system and see if there is a relationship between the length of a person's foot and the student's height.

3. Write equations or inequalities for given problem situations.

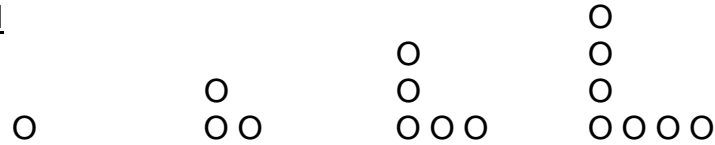
Example: Jorge joins a club to purchase CD's of his favorite Tejano groups. There is an initial membership cost of \$10, and each CD he buys costs \$6. Write an equation to show the relationship between the number of CD's he buys and his total cost.

# DRAFT

4. Represent relationships among quantities in multiple ways.

Example: If the pattern continues to grow at the same rate as shown in the picture below, how many circles will there be in the 8<sup>th</sup> figure?

Visual



Verbal The first figure has one circle, to get the second figure you add a circle horizontally and vertically, then you add two additional circles to get to the third figure, and so on.

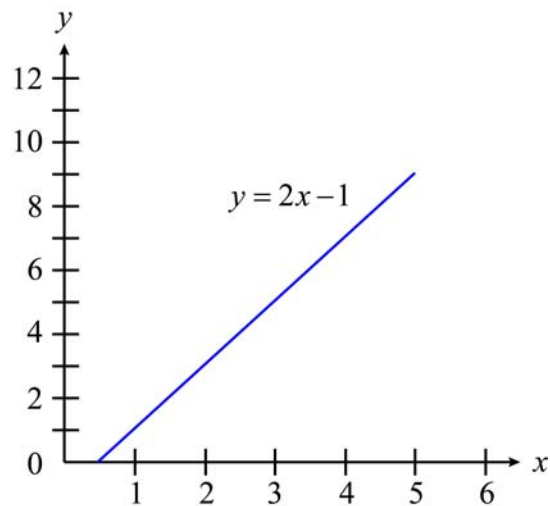
Pattern or rule If you take the number of the figure, double it, and subtract 1, then you get the number of circles.

Table

Number of figure	1	2	3	4		$n$
Number of O's	1	3	5	7		?

Ordered Pairs (1,1), (2,3), (3,5), (4,7), ...

Graph



Equation  $y = 2x - 1$

Function notation  $f(x) = 2x - 1$

# ***DRAFT***

5. Interpret and make inferences from functional relationships.

Example: The Rodriguez family chooses to send \$1 out of every \$10 they earn to their parents. If they earned \$350 this week, how much would they send to their parents?

## Assessment for Objective 1

### General strategies for assessment:

1. allow students frequent opportunities to demonstrate mastery in a variety of ways;
2. provide sufficient time for ELL students to complete assessment tasks;
3. use assessment results to design instructional planning for remediation if needed;
4. assign projects for students to work together with their partners;
5. have students write their thoughts and problem-solving actions in a journal;
6. design performance measures with visuals to check concept understanding;
7. design assessments to measure mathematical understanding, not reading comprehension;
8. ensure assignments are as free of bias as possible; and
9. make assignments that require writing explanations in English.

### Specific examples for assessment

1. Have students show multiple representations of the relationships between two variables in a writing assignment for a grade. For this task, give a visual pattern and have them show other representations of the problem to include a listing, a table, a graph, a verbal description, and an algebraic equation. For performance assessment, use a rubric that considers concept understanding, approach toward solving a problem, and verbal or picture/diagram explanations. For example:

# DRAFT

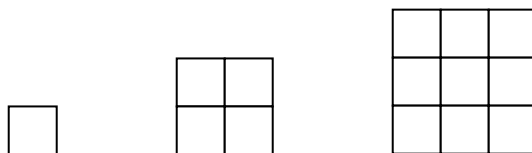
Task: Given a representation of the relationship between two variables, show other representations.

## Scoring Rubric

Conceptual Understanding	0 No evidence of understanding 1 Some understanding, but incomplete 2 Complete conceptual understanding
Approach Toward Solving Problem	0 No evidence of trying to solve problem 1 Some evidence of solving problem 2 Successful approach to solving problem
Representation of Relationship	0 No representation attempted 1 Incomplete representation 2 Complete, accurate representation

Example assessment task:

Marcella wants to build a swimming pool and she wants it to be square. Use color tiles to represent the possible areas of the pool. Assuming that each color tile represents a 1 foot by 1 foot square, use the manipulatives to model the pools, draw pictures of possible pool sizes, and look for a pattern as the pools get larger. Work with your partner and represent your patterns in a table, with ordered pairs, with a graph, and with an algebraic equation. Write an explanation in your journal to describe the area of the fifth pool in increasing size, starting with a 1 by 1 pool as the first possible pool (see drawing below). Try to determine a pattern that would help find the area of the 50<sup>th</sup> pool, without drawing all of the pools in order.





# DRAFT

2. Use traditional assessment methods, including multiple-choice questions to measure mathematics understanding also. Students need to practice solving mathematics problems in the same format of the TAKS test questions. When discussing these problems in class, have students analyze why one answer is correct and the others are incorrect. A sample problem could be:

Diego likes to ride his bike at least 20 miles every weekend, since he is training for a distance riding competition that is 100 miles long. He averages 9 miles per hour on his rides. What is the dependent variable for this functional relationship?

- a. the number of miles he rides
- b. his speed
- c. the number of hours he rides
- d. whether he finishes the 100-mile race or not

Additional problems can be found on the Texas Education Agency (TEA) website ([www.tea.state.tx.us](http://www.tea.state.tx.us)) from the TAKS information booklets ([www.tea.state.tx.us/student.assessment/taks/booklets](http://www.tea.state.tx.us/student.assessment/taks/booklets)) and from TAKS released tests ([www.tea.state.tx.us/student.assessment/resources/release/taks/index.html](http://www.tea.state.tx.us/student.assessment/resources/release/taks/index.html))

Also on the TEA website, there is a link to the TAKS Study Guide for Grade 11 Exit Level Mathematics and Science: A Student and Family Guide, which explains the key concepts under each objective and gives examples. There are additional multiple-choice problems for each objective in this guide. Although it is not designed especially for ELL students, it is a very helpful resource in preparing to take the TAKS test.

3. Design projects that involve the families of students. Have them collect data as homework assignments. For example:

Have students extend the measurement activity that was done in class. Students can measure the foot size (in centimeters or millimeters) and compare it to the height of every family member (aunts and uncles and cousins count too). Then take other measurements also, such as the length of a person's foot compared to the distance between their wrist and their elbow. In addition, compare the circumference of a family member's neck to their height. For each set of measurements, have the students verbally describe what they did, what they found, and what patterns they discovered. Patterns should be represented in a variety of ways, including by a table, a list, a graph, and an equation.

4. Let students work with a partner in class. Provide extra time for students to talk together about an assigned mathematics problem, decide how to approach it, and make a summary paper on the problem. Provide a format sheet for their mini-reports, such as a) what is the problem, b) what do we need to find out, c) how do we get started, c) how do we solve the problem, d) what is our solution, and e) how can I describe what we did? Allow time for both partners to discuss their report together, before presenting it to the class. Grade responses with a rubric that measures more than just the solution to the problem.

# ***DRAFT***

5. Make sure that all students have the resources available to accomplish every assignment. For example, do not assign projects that involve working on the Internet as a homework assignment, since not every student has access to a computer at home. If you want students to measure something, provide them with rulers or the tools to do the measuring. If you assign a project that requires the use of graphing calculators, provide the calculators and make it an in-class assignment, since many students will not have access to graphing calculators outside the classroom.