

Fractions for ELL Learners



DR. BETH BOS
TEXAS STATE UNIVERSITY
MELL CONFERENCE
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Task: Using the tools provided, come up with as many different ways to model $2/3$.

Open Number Line



Creating a Fraction Kit



Task:

You will create your own fraction kits. You will be creating one whole, halves, thirds, fourths, fifths, sixths, eighths, tenths, and twelfths.

As you are completing this task, think about what children need to know and understand to be able to make their own kits.

half = medio

fourth = cuarto 

third = tercera

fifth = quinto

sixth = sexto

seventh = séptimo

eighth = octavo

ninth = noveno

tenth = décimo

eleventh = undécimo

twelfths = duodécimo



Consider ways to reason with benchmarks when comparing these fractions.

- $5/7$ or $3/7$
- $3/8$ or $3/4$
- $5/4$ or $8/9$
- $15/16$ or $9/10$
- $1 \frac{1}{3}$ or $6/3$



Conceptual Thought Patterns



- More of the same-size parts.
- Same number of parts but different sizes.
- More or less than one-half or one whole.
- Distance from one-half or one whole (residual strategy—What's missing?)

$\frac{1}{2}, 1, 0,$



$\frac{3}{8}$

$\frac{3}{10}$

$\frac{6}{5}$

$\frac{7}{47}$

$\frac{7}{100}$

$\frac{25}{26}$

$\frac{7}{15}$

$\frac{13}{24}$

$\frac{14}{30}$

$\frac{16}{17}$

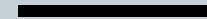
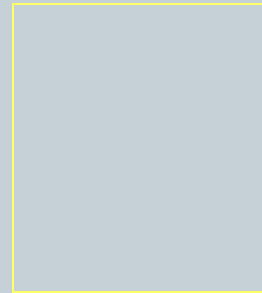
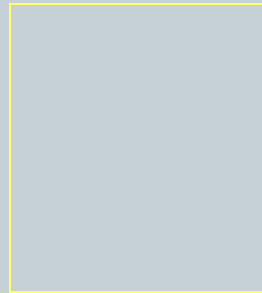
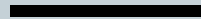
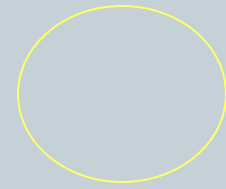
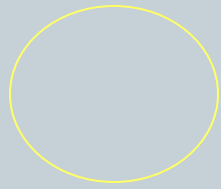
$\frac{11}{9}$

$\frac{5}{3}$

$\frac{8}{3}$

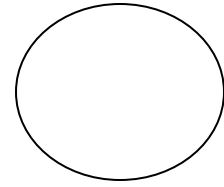
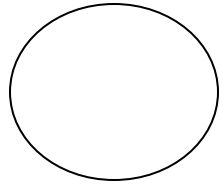
$\frac{17}{12}$

Recording Sheet for Comparing

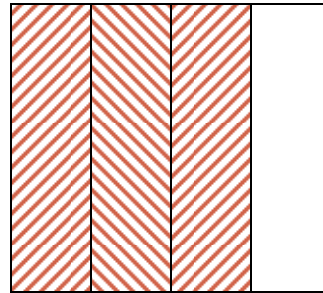


Compare

$\frac{1}{2}$ *and* $\frac{3}{4}$



$$\frac{1}{2}$$



$$\frac{3}{4}$$

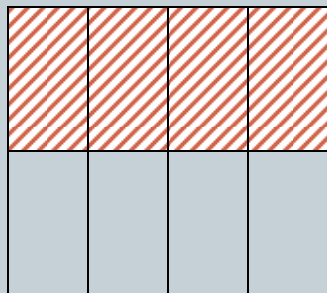
Compare

4

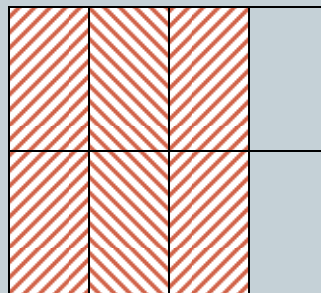
$\frac{1}{2}$ and $\frac{3}{4}$

6

$$\frac{1}{2}$$



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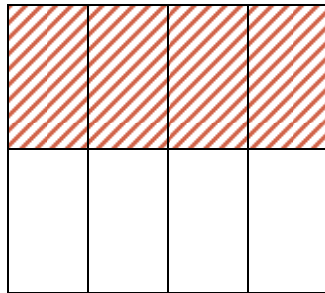
$$\frac{3}{4}$$

By how much?

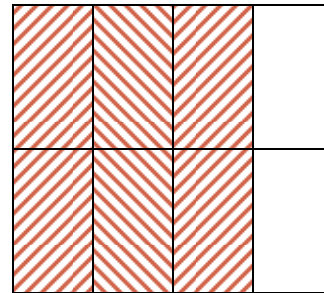
4

$$\frac{3}{4} - \frac{1}{2} = \frac{2}{8}$$

6



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We can see that the size of this difference is 2 pieces of the newly doubled cut cake. Now all we need to do is to decide how to record this. Each cake is now shared with a total 8 shares. Using “share with” and “take” we have.

$$\frac{3}{4} - \frac{1}{2} = \frac{2}{8}$$

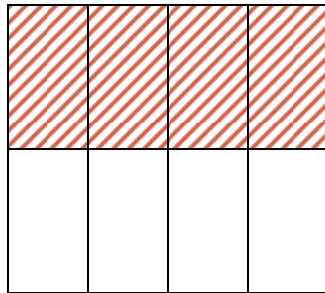
How much do we have in all?

4

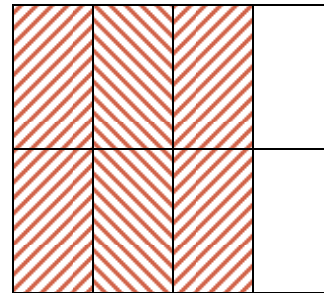
$$\frac{3}{4} + \frac{1}{2}$$

6

$$\frac{1}{2}$$



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$$\frac{3}{4}$$

We can see a total of **10** pieces of the newly double cut cake. We have **1** full cake and **2** pieces left over. To record this using the fraction symbol we can record “share with” and “take” as follows:

$$\frac{3}{4} + \frac{1}{2} = \frac{10}{8}$$

1. $\frac{2}{3}$ $\frac{1}{2}$

2. $\frac{1}{4}$ $\frac{2}{3}$

3. $\frac{2}{5}$ $\frac{1}{2}$

4. $\frac{1}{3}$ $\frac{3}{4}$

5. $\frac{3}{5}$ $\frac{2}{3}$

Divide your paper into four sections.

a. Comparison

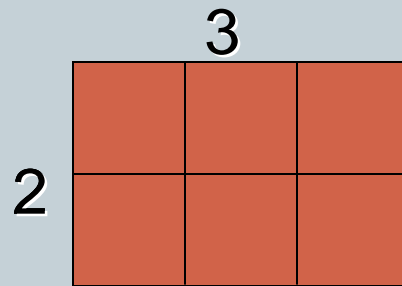
b. Addition

c. Subtraction (subtract the smaller from
larger number)

d. Word Problem

Multiplication

- Area Model: Let's look at the example of $2 \times 3 = 6$. The sketch of this would be a rectangle 2 units in one direction and 3 in the other giving rise to an area of 6 square units. This would look like:



$$2 \times 3 = 6$$

$$3 \times 2 = 6$$

$$6/3 = 2$$

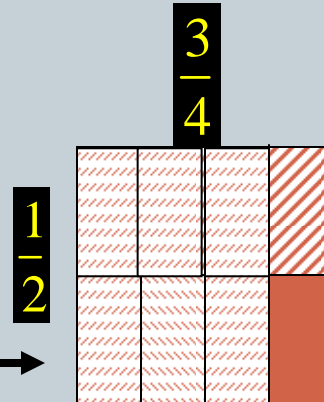
$$6/2 = 3$$

This shows how this single sketch can be used to represent the relationships between and among four related facts (fact families).

- The sketch of 2×3 showed a rectangle 2 units in one direction and 3 in the other. In a like fashion should show us a rectangle in one direction and n in the other. First look at the $\frac{1}{2}$.



Now
draw $\frac{3}{4}$



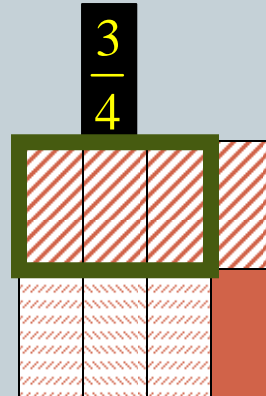
$$\frac{1}{2} \times \frac{3}{4}$$

$$\frac{1}{2}$$



Now
draw the
shared
portion

$$\frac{1}{2}$$



From the picture there are 3 small pieces which are found in the rectangle that is $\frac{1}{2}$ in one direction and $\frac{3}{4}$ in the other. We can also see that there are now a total of 8 pieces making up the full cake. So the total area we are describing in this multiplication problem is

. The number of small pieces we have, i.e. the numerator of our answer can be described by the rectangle that is 1×3 . The total size of the cake i.e. the denominator of our answer, is described by the rectangle that is 2×4 . This is identical to the procedure of multiplying *numerator x numerator* and *denominator x denominator*.

Practice



1. $3 \bullet \frac{2}{3}$

2. $\frac{3}{4} \bullet \frac{2}{5}$

3. $2\frac{1}{2} \bullet \frac{2}{3}$

How much in each group?

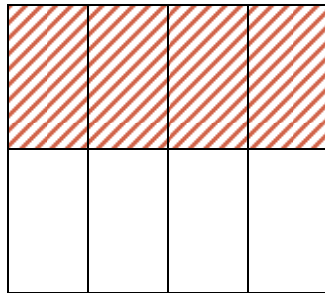
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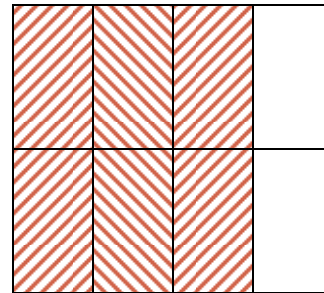
$$\frac{1}{2} \div \frac{3}{4}$$

6

$$\frac{1}{2}$$



<



$$\frac{3}{4}$$

How many $\frac{3}{4}$ are in $\frac{1}{2}$. We know that $\frac{3}{4}$ is larger than $\frac{1}{2}$ so it will go in less than one.

How much in each group?

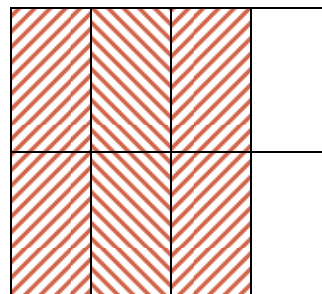
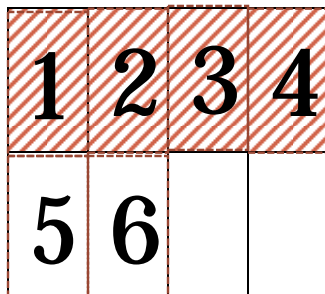
4



$$\frac{1}{2} \div \frac{3}{4}$$

6

$$\begin{array}{r} 1 \\ \hline 2 \end{array}$$



$$\begin{array}{r} 3 \\ \hline 4 \end{array}$$

How many $\frac{3}{4}$ are in $\frac{1}{2}$. We know that $\frac{3}{4}$ is larger than $\frac{1}{2}$ so it will go in less than one. Therefore we have $\frac{4}{6}$.

Practice



1. $\frac{2}{4} \div \frac{1}{4}$

2. $2\frac{1}{3} \div \frac{2}{3}$

3. $\frac{3}{4} \div \frac{1}{8}$

4. $2\frac{3}{4} \div \frac{2}{3}$

What is Relational Thinking?



Children using relational thinking draw upon their knowledge of fundamental properties of number, operations and equations to analyze a problem in the context of a goal structure and then to simplify their progress towards this goal.

For example



$$\frac{3}{4} + \frac{1}{2}$$

THINK

$$\frac{3}{4} + \frac{1}{4} + \frac{1}{4} = 1 \frac{1}{4}$$

Where

$$7a + 4a = (7 + 4)a = 11a$$



$$7/5 + 4/5 =$$

$$7 \text{ ⌚ } 1/5 + 4 \text{ ⌚ } 1/5 = (7 + 4) \text{ ⌚ } 1/5$$

$$11/5$$

**Two candy bars are shared
among three children..**



**Three one-thirds make a whole candy bar
and that one candy bar divided among
three people yields one-third of a candy
bar to each.**

$$2 + 3 = (1 + 1) + 3$$

$$1/3 + 1/3 = 2/3$$

Jeremy is making cupcakes. He wants to put $\frac{1}{2}$ a cup of frosting on each cupcake. If he makes 4 cupcakes for his birthday party, how much frosting will he use to frost all of the cupcakes?



Holly: four halves

Mr. W has 10 cups of frog food. His frogs eat $\frac{1}{2}$ a cup of frog food a day. How long can he feed his frogs before his food runs out?



John represented each cup of frog food with a rectangle, then divided each rectangle in half and notated “ $\frac{1}{2}$ ” on each half to show how much food Mr. W’s frogs could eat in a day. He then counted these to arrive an 20 days.

It takes _____ of a cup of sugar to make a batch of cookies. I have $5 \frac{1}{2}$ cups of sugar. How many batches of cookies can I make?



The choices were $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, and $\frac{3}{8}$.

8 groups of $\frac{3}{8}$ is 3 cups
4 groups of $\frac{3}{8}$ is $1\frac{1}{2}$ cups
1 more cup needed



$$\text{If } 8 \times \frac{3}{8} = 3$$

$$\frac{1}{3} \times (8 \times \frac{3}{8}) = \frac{1}{3} \times 3$$

$$(\frac{1}{3} \times 8) \frac{3}{8} = 1$$

$$\frac{8}{3} \times \frac{3}{8} = 1$$

$$3\text{cups} + 1\frac{1}{2}\text{ cups} + 1\text{ cup}$$

$$(8\text{groups} + 4\text{groups} + \frac{8}{3}\text{groups}) \frac{2}{3}$$

$$\frac{14}{3} \times \frac{3}{8}$$

Two thirds of a bag of coffee weighs 2.7 pounds.
How much would a whole bag of coffee weigh?



