

Model Lesson

Unit Name: Distance Formula		Unit Length: 1 Week	
Overview: Review of Pythagorean Theorem, derivation of distance formula from Pythagorean Theorem, use of distance formula to solve real-world problems.			
DESIRED RESULTS			
TEKS and SEs			
(8.9) Measurement. (A) (A.1) Foundations for functions. (D) (A.3) Foundations for functions. (A), (B) (A.6) Linear functions. (A)			
Enduring Understandings (Big Ideas) The Pythagorean Theorem (PT) formula can be used to solve problems connected to everyday life experiences. Students need to connect this Theorem with the formula to determine the Distance between 2 points on the Cartesian Plane. Then they can solve real world problems by selecting or developing an appropriate problem solving strategy and evaluating their problem solving method for reasonableness.		Essential Questions What are the applications of the Pythagorean Theorem? What are the properties of special right triangles and how are they used? How are the properties and relations of geometric figures important to problem solving? Given two points on a Cartesian Plane, how does the Pythagorean Theorem relate to the distance between the two points?	
Learning Goals Identify appropriate steps used to resolve unknown sides of a right triangle. Express the square root function as an exponential function. Algebraically derive the distance		Critical Vocabulary Pythagorean Theorem Distance Formula Vertex & Vertices	
Materials Needed Paper Grid paper Pencil			

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formula using principles of geometry. Use the Pythagorean Theorem to determine the distance between two points on a grid.

ASSESSMENT PLAN

Performance Tasks

On a unit test, have students determine the distances between several pairs of points. Have students demonstrate in class the application of the Pythagorean Theorem to determine the distance between two points on a grid.

Other Evidence

During the lesson, walk around and check student work to ensure understanding. During the lesson, check for understanding by using probing questions directed at individual students.

GENERAL MELL CONNECTIONS THAT APPLY TO ALL LESSONS

(MC-01) - To assist the ELLs, the teacher should avoid speaking too rapidly and when possible use basic words rather than unfamiliar ones to introduce new concepts.

(MC-02) - Teachers should foster trusting relationships with ELLs through informal conversations and presentation of a culturally rich classroom.

(MC-03) - ELLs may need extra opportunities to demonstrate mastery. Grading policies should be flexible enough to provide multiple learning opportunities without severe grade penalties.

(MC-04) - ELLs may work at a slower pace than other students because of limited English language skills and should be provided with shortened assignments, or when appropriate, extra time to work on assignments.

(MC-05) - ELLs should be scheduled in a math class that has students who have some proficiency in both languages. Teachers may need to work with counselors and others to ensure that this happens.

(MC-06) - Teachers should offer tutoring as frequently as possible and encourage ELLs to come in for extra assistance. If possible, the teacher should arrange for an aide or parent volunteer who speaks the language of the ELL to help with translation during the tutoring period on a regular schedule (for example, on Tuesdays and Thursdays, after school).

LEARNING PLAN

This lesson depends upon students' knowledge of and ability to use the Pythagorean Theorem. The first part of the lesson should focus on reviewing the Pythagorean Theorem and helping students recall its derivation and uses.

After the brief review of critical prerequisite knowledge and skills, the teacher should use modeling, demonstrations, individual and group probes to check for understanding, and guided practice to help students learn the new concept.

Review:

1. Display a right triangle on a grid, and by using probing questions, lead the students to demonstrate that if a and b are two legs of a right triangle and c is the hypotenuse, then $a^2 + b^2 = c^2$. (MC-07), (MC-09)
2. Lead the class to solve the Pythagorean Theorem for c . $c = \text{SQRT}[a^2 + b^2]$.

Lesson (During each step of the lesson, the teacher uses focused questioning to check for understanding) (MC-16):

3. Display a right triangle on a grid with the 90-degree vertex at $(0,0)$ on the grid. , (MC-09)
4. Identify each vertex of the triangle with an ordered (x,y) pair, and demonstrate that the length of the legs along the x - and y -axes corresponds to a and b in the Pythagorean Theorem and that the leg opposite the right angle corresponds to c in the PT.

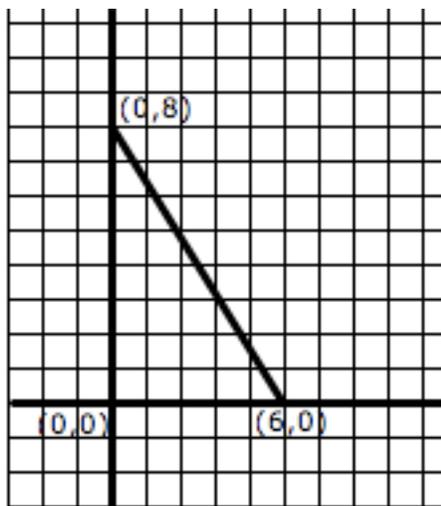
MELL Connections

(MC-07) - New concepts should first be presented to ELLs in a concrete manner, followed by a semi-concrete manner, and finally an abstract manner.

(MC-08) - Hands-on activities involving math manipulatives are typically helpful to ELLs because the lesson involves multiple learning modalities and does not require the student to rely solely upon his/her ability to understand verbal instruction.

(MC-09) - To assist the ELLs, the teacher should model the expected task and use visual representations to reinforce concepts and/or steps in the problem-solving process. Critical concepts should

(MC-09)



5. Demonstrate that the length of each leg is determined by subtracting corresponding coordinate values, i.e., the length of the leg along the x-axis is $6 - 0 = 6$, and the length of the leg along the y-axis is $8 - 0 = 8$ (Later in the lesson, demonstrate that reversing the order of subtraction does not change the result, i.e., $6 - 0$ or $0 - 6$ will yield the same result since distance on a line (one coordinate geometry) is measured by absolute value and therefore must be a positive number). (MC-09)
6. Lead a discussion and probe as necessary to demonstrate that the length of the hypotenuse is found by applying the PT. (MC-13)
7. Now, demonstrate that this example can be generalized into the distance formula: If d represents the distance from one vertex of the hypotenuse to the other, then: (MC-09)

$$d = \text{SQRT}[(x_1 - x_2)^2 + (y_1 - y_2)^2]$$

be clearly emphasized and repeated.

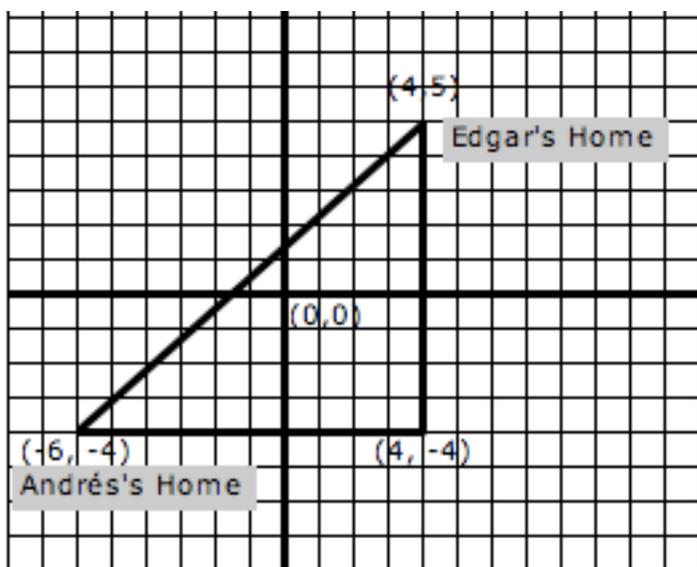
(MC-13) - Before asking ELLs to speak on a mathematics topic or problem in class, give the student time to practice what they will say with a peer tutor or partner. Without this technique, ELLs may tend to just say "I don't know" when asked a question to avoid possible embarrassment over language deficiencies.

(MC-15) - In forming groups, the teacher should make sure that ELLs are assigned with a peer tutor, that ELLs are distributed among groups, and that no group is predominantly comprised of ELLs.

(MC-16) - When monitoring ELLs during instruction, the teacher should make a special effort to assist, re-explain and demonstrate again, if necessary. Encouragement and reinforcement should be used frequently.

(MC-17) - Examples that are relevant to the lives of ELLs are helpful in motivating students and in promoting their engagement with the content.

8. Now demonstrate that the right triangle vertex need not be at (0,0) on the coordinate plane (both coordinates can be non zero). Place a right triangle with its vertex at some point other than (0,0), and demonstrate the application of the distance formula. (To create possible increased interest, use an example of wanting to find the distance from one classmate's home to another classmate's home.). (MC-09), (MC-17)



9. Apply the distance formula to obtain the distance from Edgar's home to Andrés's home: (MC-09), (MC-16), (MC-17)

$$d = \text{SQRT}[(4 - -6)^2 + (5 - -4)^2]$$

(This is a good place to demonstrate that reversing the order of the numbers to be squared does not change the result, i.e., $(4 - -6)^2 = (-6 - +4)^2$ (due to the squaring effect used to determine distance between points for Cartesian Plane measurements (2 coordinate geometry)).

$$d = \text{SQRT}[(10)^2 + (9)^2]$$

$$d = \text{SQRT}[181] = 13.45$$

<p>10. Put students in small groups or pairs, and have them apply the distance formula to several examples provided. Move about the room to check for understanding. (MC-08), (MC-15), (MC-16)</p> <p>11. Ask a student in each group or pair to explain to his/her partner(s) the process for applying the distance formula. (MC-13)</p>	
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English Language Proficiency Standards Quick Reference.
(Chapter 74. Curriculum Requirements Subchapter A. Required Curriculum, §74.4. English Language Proficiency Standards). The standards checked here are merely examples for the teacher's consideration for inclusion in this lesson.

Cross-curricular second language acquisition/ listening. The ELL listens to a variety of speakers including teachers, peers, and electronic media to gain an increasing level of comprehension of newly acquired language in all content areas. ELLs may be at the beginning, intermediate, advanced, or advanced high stage of English language acquisition in listening. In order for the ELL to meet grade-level learning expectations across the foundation and enrichment curriculum, all

- [X] (c)(2)(A) - distinguish sounds and intonation patterns of English with increasing ease;
- (c)(2)(B) - recognize elements of the English sound system in newly acquired vocabulary such as long and short vowels, silent letters, and consonant clusters;
- [X] (c)(2)(C) - learn new language structures, expressions, and basic and academic vocabulary heard during classroom instruction and interactions;
- [X] (c)(2)(D) - monitor understanding of spoken language during classroom instruction and interactions and seek clarification as needed;
- [X] (c)(2)(E) - use visual, contextual, and linguistic support to enhance and confirm understanding of increasingly complex and elaborated spoken language;
- (c)(2)(F) - listen to and derive meaning from a variety of media such as audio tape, video, DVD, and CD ROM to build and reinforce concept and language attainment;
- [X] (c)(2)(G) - understand the general meaning, main points, and important details of spoken language ranging from situations in which topics, language, and contexts are familiar to

<p>instruction delivered in English must be linguistically accommodated (communicated, sequenced, and scaffolded) commensurate with the student's level of English language proficiency. The student is expected to:</p>	<p>unfamiliar; (c)(2)(H) - understand implicit ideas and information in increasingly complex spoken language commensurate with grade-level learning expectations; and [X] (c)(2)(I) - demonstrate listening comprehension of increasingly complex spoken English by following directions, retelling or summarizing spoken messages, responding to questions and requests, collaborating with peers, and taking notes commensurate with content and grade-level needs.</p>
<p><i>Cross-curricular second language acquisition/speaking.</i> The ELL speaks in a variety of modes for a variety of purposes with an awareness of different language registers (formal/informal) using vocabulary with increasing fluency and accuracy in language arts and all content areas. ELLs may be at the beginning, intermediate, advanced, or advanced high stage of English language acquisition in speaking. In order for the ELL to meet grade-level learning expectations across the foundation and enrichment curriculum, all instruction delivered in English must be linguistically</p>	<p>[X] (c)(3)(A) - practice producing sounds of newly acquired vocabulary such as long and short vowels, silent letters, and consonant clusters to pronounce English words in a manner that is increasingly comprehensible; (c)(3)(B) - expand and internalize initial English vocabulary by learning and using high-frequency English words necessary for identifying and describing people, places, and objects, by retelling simple stories and basic information represented or supported by pictures, and by learning and using routine language needed for classroom communication; [X] (c)(3)(C) - speak using a variety of grammatical structures, sentence lengths, sentence types, and connecting words with increasing accuracy and ease as more English is acquired; [X] (c)(3)(D) - speak using grade-level content area vocabulary in context to internalize new English words and build academic language proficiency; [X] (c)(3)(E) - share information in cooperative learning interactions; [X] (c)(3)(F) - ask and give information ranging from using a very limited bank of high-frequency, high- need, concrete vocabulary, including key words and expressions needed for basic communication in academic and social contexts, to using abstract and content-based vocabulary during extended speaking assignments;</p>

<p>accommodated (communicated, sequenced, and scaffolded) commensurate with the student's level of English language proficiency. The student is expected to:</p>	<p>[X] (c)(3)(G) - express opinions, ideas, and feelings ranging from communicating single words and short phrases to participating in extended discussions on a variety of social and grade-appropriate academic topics; (c)(3)(H) - narrate, describe, and explain with increasing specificity and detail as more English is acquired; (c)(3)(I) - adapt spoken language appropriately for formal and informal purposes; and</p> <p>[X] (c)(3)(J) - respond orally to information presented in a wide variety of print, electronic, audio, and visual media to build and reinforce concept and language attainment.</p>
<p><i>Cross-curricular second language acquisition/reading</i> . The ELL reads a variety of texts for a variety of purposes with an increasing level of comprehension in all content areas. ELLs may be at the beginning, intermediate, advanced, or advanced high stage of English language acquisition in reading. In order for the ELL to meet grade-level learning expectations across the foundation and enrichment curriculum, all instruction delivered in English must be linguistically accommodated (communicated, sequenced, and scaffolded)</p>	<p>(c)(4)(A) - learn relationships between sounds and letters of the English language and decode (sound out) words using a combination of skills such as recognizing sound-letter relationships and identifying cognates, affixes, roots, and base words; (c)(4)(B) - recognize directionality of English reading such as left to right and top to bottom;</p> <p>[X] (c)(4)(C) - develop basic sight vocabulary, derive meaning of environmental print, and comprehend English vocabulary and language structures used routinely in written classroom materials;</p> <p>[X] (c)(4)(D) - use prereading supports such as graphic organizers, illustrations, and pretaught topic-related vocabulary and other prereading activities to enhance comprehension of written text;</p> <p>[X] (c)(4)(E) - read linguistically accommodated content area material with a decreasing need for linguistic accommodations as more English is learned;</p> <p>[X] (c)(4)(F) - use visual and contextual support and support from peers and teachers to read grade-appropriate content area text, enhance and confirm understanding, and develop vocabulary, grasp of language structures, and</p>

<p>commensurate with the student's level of English language proficiency. For Kindergarten and Grade 1, certain of these student expectations apply to text read aloud for students not yet at the stage of decoding written text. The student is expected to:</p>	<p>background knowledge needed to comprehend increasingly challenging language;</p> <p>(c)(4)(G) - demonstrate comprehension of increasingly complex English by participating in shared reading, retelling or summarizing material, responding to questions, and taking notes commensurate with content area and grade level needs;</p> <p>(c)(4)(H) - read silently with increasing ease and comprehension for longer periods;</p> <p>[X] (c)(4)(I) - demonstrate English comprehension and expand reading skills by employing basic reading skills such as demonstrating understanding of supporting ideas and details in text and graphic sources, summarizing text, and distinguishing main ideas from details commensurate with content area needs;</p> <p>[X] (c)(4)(J) - demonstrate English comprehension and expand reading skills by employing inferential skills such as predicting, making connections between ideas, drawing inferences and conclusions from text and graphic sources, and finding supporting text evidence commensurate with content area needs; and</p> <p>[X] (c)(4)(K) - demonstrate English comprehension and expand reading skills by employing analytical skills such as evaluating written information and performing critical analyses commensurate with content area and grade-level needs.</p>
<p><i>Cross-curricular second language acquisition/writing</i> . The ELL writes in a variety of forms with increasing accuracy to effectively address a specific purpose and audience in all content areas. ELLs may be at the</p>	<p>[X] (c)(5)(A) - learn relationships between sounds and letters of the English language to represent sounds when writing in English;</p> <p>[X] (c)(5)(B) - write using newly acquired basic vocabulary and content-based grade-level vocabulary;</p> <p>[X] (c)(5)(C) - spell familiar English words with increasing accuracy, and employ English spelling patterns and rules with increasing accuracy as more English is acquired;</p>

<p>beginning, intermediate, advanced, or advanced high stage of English language acquisition in writing. In order for the ELL to meet grade-level learning expectations across foundation and enrichment curriculum, all instruction delivered in English must be linguistically accommodated (communicated, sequenced, and scaffolded) commensurate with the student's level of English language proficiency. For Kindergarten and Grade 1, certain of these student expectations do not apply until the student has reached the stage of generating original written text using a standard writing system. The student is expected to:</p>	<p>(c)(5)(D) - edit writing for standard grammar and usage, including subject-verb agreement, pronoun agreement, and appropriate verb tenses commensurate with grade-level expectations as more English is acquired;</p> <p>[X] e(c)(5)(E) - employ increasingly complex grammatical structures in content area writing commensurate with grade-level expectations, such as:</p> <ul style="list-style-type: none"> (i) using correct verbs, tenses, and pronouns/antecedents; (ii) using possessive case (apostrophe s) correctly; and (iii) using negatives and contractions correctly; <p>(c)(5)(F) - write using a variety of grade-appropriate sentence lengths, patterns, and connecting words to combine phrases, clauses, and sentences in increasingly accurate ways as more English is acquired; and</p> <p>[X] (c)(5)(G) - narrate, describe, and explain with increasing specificity and detail to fulfill content area writing needs as more English is acquired.</p>
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Appendix 1
TEKS and Student Expectations

(8.9) **Measurement.** The student uses indirect measurement to solve problems.

The student is expected to: (A) use the Pythagorean Theorem to solve real-life problems.

(A.1) **Foundations for functions.** The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student is expected to:

(D) represent relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities.

(A.3) **Foundations for functions.** The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

The student is expected to:

(A) use symbols to represent unknowns and variables; and
(B) look for patterns and represent generalizations algebraically.

(A.6) **Linear functions.** The student understands the meaning of the slope and intercepts of the graphs of linear functions and zeros of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student is expected to:

(A) develop the concept of slope as rate of change and determine slopes from graphs, tables, and algebraic representations.