Mathematics and English Language Learners in High School: A Review of the Literature

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Executive Summary

The *No Child Left Behind Act* (NCLB) has mandated that all children be provided a learning environment in which students are taught by licensed professional teachers utilizing research-based best practices in schools that make annual yearly progress toward the success of every student in academic endeavors.

It is well documented that for some segments of our national student population, this lofty ideal is not being reached. This is the case for some Hispanic students (and students of other nationalities with native languages different from English) in general and for English Language Learners in particular. English Language Learners (ELL) are, from the time they enter a U. S. public school, challenged (1) to learn a new language, (2) to learn a new language in a relatively short time span, (3) to learn and master the content of at least the core disciplines, (4) to pass state-wide high stakes testing at periodic points along their educational career, and (5) to pass state-wide high stakes testing at the end of their educational career in order to receive a high school diploma. The challenge for teachers and administrators is to provide a positive learning environment that successfully maximizes the learning experiences of ELL students and provides them access to the opportunities for other educational experiences and meaningful participation in the democratic experience.

In order to provide a background perspective for this report, it should be noted that nationally there are approximately 5.5 million non-English speaking students in U. S. public schools (2004). Of those students, 440 different languages are spoken among our ELL population. Of those different languages, 80% speak Spanish. Closer to home in 2003-2004 more than 43.8 percent of Texas public school students are Hispanic. In addition, 3.3 percent are classified “other” (that is, not African-American, White, or Hispanic) and includes other ethnic groupings. Further, 14.1 percent of Texas public school students are identified as limited English proficient. According to data collected through the Texas Education Agency (TEA) Public Education Information Management System (PEIMS) in the 2003-2004 school year, 606,190 Limited English Proficient (LEP) students (up 6.0% from the previous year) were enrolled in public schools. In the nation Texas ranks second to California in terms of number of LEP students enrolled.

According to PEIMS data for the 1999-2000 school year, more than 90 percent of Texas LEP students speak Spanish as their primary language. A small but significant number of LEP students speak a variety of other languages, with Vietnamese, Chinese, and Korean being the most prevalent. Responses to teacher surveys suggest that there are more than 100 different home languages represented in the Texas LEP student population.

The purpose of this paper was to review the literature regarding English Language Learners (ELL) and the teaching of mathematics, particularly for secondary students (11th grade).

Our initial approach was to review the literature with emphasis on research studies on ELL and mathematics. Our goal was to present these findings and provide a stepping stone to identify best practices for the teaching of mathematics to ELL students and to create resources for teachers to access that would facilitate his/her professional growth in addressing the needs—instructionally and otherwise—of ELL students in Texas classrooms.

The number of research studies specifically addressing ELL and mathematics was extremely limited. There were few studies (1) that were research-based and (2) that had as a focus the teaching and learning of mathematics for secondary ELL students. Studies typically available addressed mathematics and elementary ELL students rather than secondary students. Furthermore, the majority of the literature on mathematics and ELL students was not research-based as we hoped to find, but rather addressed the topics of literacy and mathematics together as a subsection of literacy development for ELL students in general. Going even further, we found that the literature of mathematics and ELL students was but a subsection of the overall bilingual/language acquisition discussion. In the current literature it seems you cannot talk about mathematics without talking about literacy and second language acquisition.

We began our search with a wide net. We reviewed over 300 articles with any connection to "mathematics" and "ELL". From there we identified studies—those with at least an identified sample and some experimental procedure (quantitative studies) or description of a successful program (qualitative studies) connected to "mathematics" and "ELL students" regardless of grade level. We reviewed reports from established organizations (e.g., Center for Research on Education, Diversity, and Equity, National Clearinghouse on Bilingual Education, and the Center for Applied Linguistics) and symposia proceedings, particularly if they served bilingual, ELL, or second language acquisition populations. We reviewed websites, e-documents, homepages of organizations and individuals regarding ELL research or practice. Finally, we attended conferences on ELL students just to make sure that we were on the right track in our understanding of the literature and identification of current practice among teachers and other professionals.

The result of our search of the literature was to present the following findings and recommendations. In addition, we were specifically charged to identify what was missing from the literature. In honoring this request we have provided topics that were "hinted at" and that were clearly missing in considering the topic of mathematics and ELL students.

Findings Based on a Search of the literature:

- There is a paucity of research specific to mathematics and ELL students in general and for secondary ELL students in particular.
- In the current literature on mathematics and ELL students, the consensus is that mastery of content, the principles of literacy, and language acquisition are tied together—content, literacy, and language acquisition go hand-in-hand.
- Two-way dual language programs hold the most promise for delivery of all content to ELL students; one way language programs are the second most successful; all others fail to reach the 50% mark (that is, the mean score for all groups compared) and decline over time as well (see Thomas & Collier, 2002).
- Teachers (and teacher preparation programs) must acknowledge that mathematics has a formal content language register and the teaching of this register is critical to all students' understanding of math, beginning in the elementary school.
- The inability of all types of students to successfully translate word problems is a universal source of frustration for mathematics instruction in K-16, and is not just limited to ELL students; however, the language barrier of ELL students compounds their lack of understanding.
- Mathematics scores statewide are below acceptable levels in *all* student categories, particularly at Algebra I. This would seem to accentuate the existence an overall problem in mathematics instruction.

Findings Hinted at, but Not Discussed in the Literature:

- The compartmentalization of content, in this case mathematics, so that students fail to see connections across subject areas; this is particularly true in secondary education environments and in the case of ELL students.
- Teaching for understanding as a goal of instruction for all content areas. Teaching for understanding is inhibited by the previous point, but should be a recognized goal of instructional efforts at all grade levels.
- Teachers, particularly elementary teachers, may not be skilled in the technical aspects of mathematics *register* and how to teach it effectively and with consistency so that it provides the needed foundation for the teaching of secondary mathematics.
- The effects of power manifestation inherent in US school classrooms—English is dominant, all other languages are secondary; thus, traditional second language acquisition programs seek to change minority students to be like the majority. The exception to this reality is found in dual

language programs, where native languages and English are seen as equally important, and all students benefit from instructional efforts in two languages.

- The role and influence of the principal in the viability and success of ELL programs.
- The role and influence of the teacher in the viability and success of ELL programs.

What is not Discussed in the Literature:

- The difference between how mathematics is taught in other countries and how it is taught in the US (i.e., conceptually-based vs. process-based; see the Third International Math & Science Studies for more support of this idea or *The Teaching Gap* by Hiebert & Stiggins).
- The acknowledgement that the poor math scores statewide in particular may be indicative of ineffective teaching strategies for all students and that the math curriculum needs to be revamped to reflect a different mode of instructional focus (i.e., cover less more deeply) and alternative instructional strategies that promote engagement, understanding, and higher order thinking.

Recommendations:

- Support and fund research to continue to identify programs that successfully tie literacy to content instruction, particularly as it concerns mathematics, such as SLAMS (Cuevas, 1981, 1984, 1992).
- Support and fund the development of Two-way dual language programs at all school levels, not just elementary schools where these programs are typically found.
- Support and fund programs that build on the positives of language acquisition for all students at all grade levels rather than programs that isolate students because of their native language and seek to change them to English speakers primarily.
- Support and fund programs to identify and implement instructional strategies that engage all students such as Instructional Conversations, Concept/thematic instructional practices, etc.
- Support and fund professional development for teachers regarding needs of ELL students and instructional practices that support ELL students (see CREDE Five Standards for Effective Pedagogy and Student Outcomes).
- Examine teacher preparation programs; require elementary teachers to have formal training in math instruction, particularly in learning the mathematics register, and/or in instructional strategies that facilitate development of literacy and mathematics skills.
- Recognize and support the importance of mathematics pedagogy in teacher preparation courses and programs. Demand for teacher mastery of mathematics is seen as a crucial to being able to teach mathematics register, concepts, and processes. However, an equal assertion can be made that there is a difference in *learning* mathematics and *teaching* mathematics. Teachers must not only understand mathematics, they must be able to *teach* in a way that students can understand mathematics. That is, teachers must receive training in mathematics pedagogy.
- Support changes in math instruction from process to conceptual foundations for all K-12 students.
- Rewrite math TEKS to support *depth* rather than *breadth* in instructional focus. Further, support the conceptual understanding of mathematics through revamping of TAKS testing of mathematics.

Mathematics and English Language Learners: A Review of the Literature

Introduction

The *No Child Left Behind Act* (NCLB) has mandated that all children be provided a learning environment in which students are taught by licensed professional teachers utilizing research-based best practices in schools that make annual yearly progress toward the success of every student in academic endeavors.

It is well documented that for some segments of our national student population, this lofty ideal is not being reached. This is the case for some Hispanic students (and students of other nationalities with native languages different from English) in general and for English Language Learners in particular. English Language Learners (ELL) are, from the time they enter a U. S. public school, challenged (1) to learn a new language, (2) to learn a new language in a relatively short time span, (3) to learn and master the content of at least the core disciplines, (4) to pass state-wide high stakes testing at periodic points along their educational career, and (5) to pass state-wide high stakes testing at the end of their educational career in order to receive a high school diploma.

The corresponding challenge for teachers and administrators is to provide an learning environment in which ELL students can (1) learn a new language, (2) learn a new language in a relatively short time span, (3) learn and master the content of at least the core disciplines, (4) pass state-wide high stakes testing at periodic points along their educational career, and (5) pass state-wide high stakes testing at the end of their educational career in order to receive a high school diploma.

Needless to say, the challenge is formidable. Not *impossible*, but *formidable*. Consider these points from the American Federation of Teachers Policy Brief 17 dated March 2004, which provided these national statistics regarding Latino/ELL students:

- *Latino* is a term that encompasses a multiracial, multinational, and diverse group in its educational and socioeconomic background; nationally the group is heterogeneous
- Despite signs of progress, educational outcomes for Latinos have not improved dramatically in the last 30 years. Latinos continue to have low academic achievement and the highest dropout rates in the nation, as well as low college preparatory course enrollment and post secondary attainment.
- According to NAEP, one third of Latino students perform below grade level. In the most recent NAEP results (NCES, 2003) only 11 percent of Latino eight graders scored at or above proficient in math, compared to 36 percent of white ninth graders. In reading, only 14 percent of Latino eighth graders scored at or above proficient, compared to 39 percent of white eight graders.
- Latinos have the highest dropout rates of any major ethnic group in the United States.
- One third of Latino students perform below grade level, which increases their chances of dropping out of school from 50 percent to 98 percent, depending on how far behind they are.
- Latinos tend to drop out earlier, between the eighth and tenth grades, than other students.

More specifically, in 2004 there were approximately 5.5 million non-English speaking students in U. S. public schools (Leos, 2005). Of those students, 440 different languages are spoken among ELL populations. Of those different languages, 80% speak Spanish. And closer to home? In 2003-2004 more than 43.8 percent of Texas public school students were Hispanic. In addition, 3.3 percent were classified “other” (that is, not African-American, White, or Hispanic) and included other ethnic groupings. Further, 14.1 percent of Texas public school students were identified as limited English proficient. According to data collected through the Texas Education Agency (TEA) Public Education Information Management System (PEIMS) in the 2003-2004 school year, 606,190 Limited English Proficient (LEP) students (up 6.0% from the previous year) were enrolled in public schools. In the nation Texas ranks second to California in terms of number of LEP students enrolled.

Similarly, consider this dated but relevant picture of Hispanic achievement presented by Crandall, Dale, Rhodes, and Spanos (1985):

> Hispanic underachievement is one of the most serious problems facing American educators. Approximately 40% of Hispanic students leave school before the 10th grade, and an additional 13% dropout before graduation. Of those who remain in high school, approximately 75% are enrolled in a vocational or general educational track. Only 30% enroll in college, usually a 2-year, open door institution (Rendon, 1983). Thus, Hispanic students comprise only 3.7% of full-time undergraduate enrollment and 2.2% if graduate enrollment in U. S. colleges and universities (U. S. Department of Commerce, 1981). Among all groups within the U. S. population, Hispanics have the lowest number of college graduates (American Council on Education, 1984). Of those who graduate from college, the majority study education, public affairs, or social sciences. Very few pursue scientific or technical careers. As Mestre, Gerace, and Lochhead (1982) point out, “the percentage of Hispanics who are enrolled in postgraduate level programs, or who receive baccalaureate degrees in technical fields, is considerably less than their overall representation in college programs.” As a consequence, Hispanics are disproportionately underrepresented in scientific and technical fields, such as physics, computer science, or engineering (National Center for Education Statistics, 1978). According to Valverde (1984), “of the 44,800 mathematicians reported in the 1970 labor force, only 637 or 1.3% had a Spanish surname, (p. 129)

While these points do not specify ELL students in particular, they do highlight the general problems challenging schools for this population of students, especially if performance in mathematics considered. For ELL students, who include not only Hispanic students but also other varieties of minority languages, the situation is even more pronounced.

Purpose

The purpose of this paper was to review the literature regarding the teaching of mathematics to secondary English Language Learners (ELL).

Our initial approach was to review the literature with emphasis on research studies on ELL and mathematics. Our goal was to present these findings and provide a stepping stone to identify best practices for the teaching of mathematics to ELL students and to create resources for teachers to access that would facilitate his/her professional growth in addressing the needs—instructionally and otherwise—of ELL students in Texas classrooms.

As our work progressed, it became readily apparent that the number of research studies specifically addressing ELL and mathematics was extremely limited. There are few studies that are research-based and that have as a focus the teaching and learning of mathematics for secondary ELL students. The studies typically available addressed mathematics and elementary ELL students rather than secondary. Furthermore, the majority of the literature on mathematics and ELL students was not research-based as we hoped to find, but rather addressed the topics of literacy and mathematics together as a subsection of literacy development for ELL students in general. Going even further, we found that the literature of mathematics and ELL students is but a subsection of the overall bilingual/language acquisition discussion. In the current literature it seems you cannot talk about mathematics without talking about literacy. And, you cannot talk about literacy without talking about language acquisition and development. So then, it seems that at present our topic is not a specific location on the grand map of the literature on language acquisition/content area learning (i.e., mathematics), but rather is more like a small ship a-sail on the vast ocean of literature—and very active if not heated discussion—on language acquisition/content area learning for non-English speakers and the multiple program approaches for addressing the instructional needs of ELL students.

We were somewhat relieved as our deliberations progressed to be redirected in our focus by the math coordinator for TEA, formerly Paula Mueller, as it was her belief that TEA had a good idea of what was “out there” regarding ELL students based on previous research efforts and projects sponsored and funded by TEA. Consequently, she voiced her interest in our discovering what was “not there” regarding mathematics and ELL students.

Given this challenge, we proceeded to follow the various strands of the literature regarding “mathematics” and “ELL students” wherever they might lead. As a result, we have attempted to present our work within the context of the various discussions on literacy, on programs servicing ELL students, on instructional practices for ELL students, and on problems in teaching mathematics. By doing so, we hope that we have identified some areas for examination—both for future research and future practice—and that instead of adding to the confusion and chaos of philosophical bantering, we might have added some bridges to the various topics surrounding the teaching of mathematics to ELL students.

Scope of this Study

We began our search with a wide net. We reviewed over 300 articles with any connection to “mathematics” and “ELL”. From there we identified studies—those with at least an identified sample and some experimental procedure (quantitative studies) or description of a successful program (qualitative studies) connected to “mathematics” and “ELL students” regardless of grade level. We reviewed reports from established organizations (e.g., Center for Research on Education, Diversity, and Equity, National Clearinghouse on Bilingual Education, and the Center for Applied Linguistics) and symposia proceedings, particularly if they served bilingual, ELL, or second language acquisition populations. We reviewed websites, e-documents, homepages of organizations and individuals regarding ELL research or practice. Finally, we attended conferences on ELL students just to make sure that we were on the right track in our understanding of the literature and identification of current practice among teachers and other professionals.

Background of LEP in Texas

According to TEA (2000), Texas schools offer special language programs to LEP students. When a student is identified as limited English proficient, the student is instructed in either a bilingual education program or a program of English as a second language (ESL). Bilingual education programs are designed to teach students the English language while providing academic instruction in their primary language until they have learned enough English to transition to academic learning in English. ESL programs are designed to teach students the English language and provide academic instruction in English using teaching methodologies that are designed specifically for second language learners. Texas law mandates that schools offer bilingual education programs in the elementary grades when the enrollment of students of one language group is at least 20 in a grade. Beyond the elementary grades, bilingual education programs are not mandated. On all elementary, middle school, or high school campuses with any LEP students, an ESL program is required if bilingual education is not offered. According to PEIMS data for the 1999-2000 school year, the number of LEP students in bilingual education programs was 276,157, and the number in ESL programs was 220,794.

LEP students are served through bilingual education or ESL programs until they are identified as English proficient based on stare-determined criteria. Once they have met the criteria, they exit these special language programs and are no longer identified as limited English proficient. Schools who have LEP students are required to establish a Language Proficiency Assessment Committee (LPAC) to identify LEP students and ensure proper program placement and exit for effective transition to mainstream English instruction. This committee is also responsible for determining whether students meet the criteria for a LEP exemption from TAAS and whether they should take TAAS in English or Spanish.

According to PEIMS data for the 1999-2000 school year, more than 90 percent of Texas LEP students speak Spanish as their primary language. A small but significant number of LEP students speak a variety of other languages, with Vietnamese, Chinese, and Korean being the most prevalent. Responses to teacher surveys administered during the development of the Reading Proficiency Test in English suggest that there are more than 100 different home languages represented in the Texas LEP student population.

Challenges to Teaching ELL

Abedi and Dietel (2004) assert that the challenges for English language learners are especially difficult and include both educational and technical issues:

1. Historically low ELL performance and slow improvement: English language learners consistently perform lower than other students and frequently lower than many other subgroups.
2. Measurement accuracy: Findings suggest that low ELL language ability decreases ELL performance on most tests, which influences the test with regard to its ability to accurately measure ELL content knowledge. Further, the test becomes a measure of two skills—*subject and language*.
3. Instability of the ELL student subgroup: Instability is caused by two influences. First, in some states, as students become language proficient, they are reclassified and the ELL subgroup scores tend to drop. Second, a continuous increase of low-achieving ELL students makes it more difficult to achieve progress in improving ELL scores.
4. Factors outside of a school's control: Nonschool factors include such variables as parent education level or socioeconomic status and tend to outweigh school factors in their effect on student achievement.

Education Resources Information Center (ERIC) Digests Relating to MELL/ELL

The Education Resources Information Center (ERIC), sponsored by the Institute of Education Sciences (IES) of the U.S. Department of Education, produces a premier database of journal and non-journal education literature, including "digests" which are summaries of educational literature, including both research (i.e., quantitative or qualitative studies) and non-research information. Two ERIC Digests are presented in this section. These documents do not relate findings on research related to mathematics and ELL students, but instead offer information for practitioners thought to be beneficial to ELL students. Included are (1) a brief description of two math programs thought to help ELL students (Schwartz), and (2) techniques for teaching ELL students in science (Sutman).

Schwartz (1991) describes two math instruction programs that hold promise for use with LEP students in bilingual and all English classrooms. The first, Active Mathematics Teaching (ATM), is a form of "direct instruction." Its function is to convey large amounts of highly structured information to students just beginning to learn a subject. Teachers using ATM in classes with LEP students are urged to facilitate learning by providing the definitions of math language in ways that students are sure to understand. The second method, Cognitively Guided Instruction (CGI), focuses on the student's thought processes while they solve math problems. Teachers are urged to take this information into account as they make instructional decisions. Further, teachers are urged to allow students to express themselves in the language they can use most comfortably. In doing so, the student's English language fluency is expected to increase.

Sutman et al (1993) discuss teaching science to ELL students. While, of course, science is not mathematics, the topics presented by these authors reflect the trend in teaching ELL students in general and add depth to our discussion. They note that a major goal of science instruction is to develop students' ability to interpret and apply what they have learned. So then, instructional techniques must stress development of thinking skills as well as acquisition of science information. These authors promote two instructional techniques.

The first technique is collaborative learning. It is asserted that research and experience have demonstrated that the classroom organization strategy most effective for teaching science to LEP students is cooperative learning because it fosters language development through inter-student (and possibly written) communication. Within this framework, students should be given ample opportunity to make choices and decisions, within the groups and personally, about how to organize their projects. They should be encouraged to evaluate their own work, to challenge each other's explanations and approaches within the group, and to discuss coursework with the teacher. Students should also be given ample opportunities to test their own ideas.

The second technique is inquiry/discovery instruction. It is asserted that each student has the opportunity to find answers to questions they have posed to themselves. Students develop language skills as they articulate the problems they have devised and strive to solve. Within this framework, teachers should pose open-ended questions to students and provide references to stimulate student thinking and to use their skills in language. The inquiry/discovery method of science teaching is like the whole or natural language approach to teaching which deemphasized memorization, stressing instead language skill development and comprehension through the use of language in a real world setting, in this case the science classroom.

Sutman et al (1993) advocate several orientations of curriculum. First, science instruction is most effective when the content is organized around common themes. This approach puts scientific knowledge in a comprehensible context with relevance to students' lives, which increases the probability that students will continue to want to learn science and language on their own; extends the time over which a single topic is studied, allowing more time for understanding and reflection, and for repetition in the use of the English vocabulary; and reduces the propensity to overcrowd the curriculum with complex content and vocabulary. Second, English language development must be an integral objective of all science instruction. It is important to include vocabulary development. Third, the integration of science and mathematics teaching provides opportunities for students to utilize skills and understandings for both subjects. As students pose and solve science problems, they will naturally require use of mathematics, so combining instruction in both subjects, along with English language skills development, reinforces learning of each.

Literature Related to Mathematics for ELL Students (MELL)

This section highlights offerings in the literature that are related to mathematics and ELL students. Two studies offered in this section are the well referenced and oft quoted works from the *Delaware Symposium on Language Studies (1985)*. The first is from the work of Crandall, Dale, Rhodes, and Spanos on the language of mathematics and offers a study of mathematics and ELL college students. The second is from Kessler, Quinn, and Hayes on relationship among mathematics, language, and second language acquisition. A third key work though not from the Delaware Symposium is from Dale and Cuevas regarding the integration of mathematics and language learning, including strategies and techniques to make teaching effective. Henderson and Landesman follow with a study of middle school students taught by thematically integrated mathematics instruction. Mather and Chiodo next provide a discussion of teaching mathematics to elementary ELL students, and they provide a list of possible strategies for teaching. Similarly, Anstrom provides a list of classroom strategies identified from his research for use with LEP students. Finally, Moschokovich explores three perspectives used to teach bilingual mathematics learners, and she offers the value and limitations of each perspective.

The *Delaware Symposium on Language Studies (1985)* provided two discussions on the challenge of teaching mathematics to ELL students.

In the first Symposium offering Crandall, Dale, Rhodes, and Spanos (1985) discussed what they believed to be the critical factor in ELL student success in mathematics—the language of mathematics. These authors make their thinking in this regard explicit:

> The role of language is ubiquitous. It is the medium by which teachers introduce and convey concepts and procedures, through which texts and problems are read and solved, and by which math achievement is measured. Language skills—particularly the reading skills needed to comprehend mathematics texts and word problems and the listening skills required to understand and follow an instructor's presentation of a problem's solution—are the vehicles through which students learn and apply math concepts and skills (p. 130).

Noting that the language of mathematics is complex, the authors present Halliday's (1975) idea that mathematics has its own register and includes several types of vocabulary characteristic of the mathematics register:

1. Words which have a different meaning in the context of mathematics than in natural language, such as *sum, even, add, equal, divide*.

2. Technical vocabulary often formed from Greek and Latin words, such as *exponent, equilateral, triangle, hypotenuse*.
3. Terms created from combinations of natural language words, such as *prime number, rational number*.
4. Complex strings of words such as *least common denominator* or least common multiple.

The authors also note that mathematics language poses problems for students at the discourse level—the meaningful chunks of math language or *texts* that need to be interpreted both sentence by sentence and in terms of their role in a specific context. Regarding this area, the authors highlight Bye (1975) who enumerated the following problematic features of mathematics texts:

1. They are conceptually packed.
2. They have a high-density factor.
3. They require up-and-down as well as left-to-right eye movements.
4. They require reading-rate adjustment.
5. They require multiple readings.
6. They use numerous symbolic devices.
7. They contain a great deal of technical language with precise meaning.

Using these as a base, Crandall et al designed a research project to (a) investigate linguistic features that pose difficulties in learning the basic principles of algebra, including problem solving, (b) develop materials that would address these linguistic difficulties and at the same time supplement students' acquisition of basic algebra concepts. Working with the mathematics departments of three colleges, the authors focused on developmental algebra courses in which students reviewed basic math and began working with increasingly abstract and symbolic language. The study procedures included the following:

1. Analysis of algebra texts used at each of the institutions;
2. Observations of 30 mathematics, developmental algebra, and college algebra courses;
3. Interviews with 30 mathematics faculty from the three institutions focusing on the types of difficulties that students have in algebra and on the ways in which teachers have addressed them; and
4. Interviews with 46 developmental and college algebra students, some Hispanic, some LEP, and some native English speaking.

The results of the study led to the identification of a variety of linguistic features at the lexical, syntactic, semantic, and discourse levels which cause difficulties for beginning algebra students. The language of mathematics contains several types of vocabulary: ordinary terms, technical terms, symbolic expressions, and terms with multiple meanings. Another problem is posed by the symbolic or notational language that must be mastered, in which a new set of symbols denotes, for example, *larger than* or *smaller than*. Another, perhaps more subtle aspect of math vocabulary involves the many ways in which the same math operation can be signaled (i.e., *combine, add, plus, sum, and*). Similarly, with subtraction (i.e., *subtract, minus, from, less than*). At the syntax level, students typically duplicate the surface syntax of the statement of the problem which results in the inversion of the correct algebraic statement (e.g., The number a is 5 more than the number b is often written as $a + 5 = b$ rather than $a = b + 5$). Inversion errors result from students' expectation of finding a one-to-one correspondence between the words in the problem and the algebraic equation. On the semantic level, the greatest problem involves reference. That is, students do not realize that the number referred to by *a number* and *the number* is in fact the same number (e.g., 8 times a number is 30 more than 6 times the number). The fact that numbers, operations, functions, values, and variables are abstract entities which are couched in terms of natural language in word problems causes students difficulties, particularly when ambiguity occurs in the presentation of the problem. At the discourse level, some word problems are so contrived or divorced from student experience that students cannot begin to understand what the point of the problem is, let alone set it up and solve the problem.

In the second Symposium offering Kessler, Quinn, and Hayes (1985) discussed the relationship among mathematics, language, and second-language acquisition. In particular, they focused on four areas: (a) the nature of mathematical performance, (b) the language of mathematics, (c) problems that mathematics pose for minority children learning English as a second language, (d) mathematics as a facilitator of second-language development. These authors assert that

> processing mathematics successfully rests on the ability to utilize very precise language of mathematics in doing mathematical reasoning. The context reduced language of mathematics, the extensive use of logical connectors, the specialized vocabulary and syntactical structures, and appropriate discourse rules, all present a complex set of problems for LEP children engaged in mathematics discourse (p.152).

For Kessler et al, mathematical thinking is a style of thinking that is a function of particular operations, processes, and dynamics connected to the study of mathematics. Operations include enumeration or counting, iteration or repetition of a pattern, finding relationships among elements by ordering or making correspondences, and transforming elements by combination or substitution. Processes include examining particular examples, conjecturing about any underlying pattern that may connect mathematical elements, generalizing upon recognition of a pattern or regularity, and convincing others as a result of constructing testable arguments. Dynamics include manipulating elements such as physical objects, diagrams, ideas, and symbols, as well as getting a sense of pattern or relatedness and articulating it verbally or nonverbally. In addition, cognition—the doing of mathematics—and metacognition—the choosing and planning what to do and monitoring what is being done—factor into the overall scheme as well.

Underlying the concept of mathematical thinking are the works of Cummins and Dawe on the concept of language proficiency. Cummins supposes that there are two continua illustrating language proficiency. The first continuum relates to the range of contextual support available to the language user. At one end is Basic Interpersonal Communication Skills (BICS) in which context-embedded communication relies strongly on concrete, “visible” situational cues to meaning. At the other end is Cognitive Academic Language Proficiency (CALP) in which context-reduced relies strongly on linguistic or literary-related cues to meaning. The second continuum addresses the degree of cognitive demands imposed by specific tasks or activities from demanding to undemanding. With this formulation, Cummins asserts that BICS is the language of familiarity, of home which students bring to school while CALP is the language of the disciplines studied at school, of which students may or may not be familiar and of which students must learn to master the discipline (i.e., mathematics register). Similarly, Dawe proposes a model for mathematical proficiency. He distinguishes between surface fluency in basic mathematical skills (BIMS) from cognitive, analytical, mathematical proficiency (CAMP). Independent of any specific natural language, CAMP is the ability to reason efficiently with the abstract deep structures of mathematics.

Kessler et al note that mathematics uses linguistic features specific to its discourse. To extract meaning from mathematical statements and to convey that meaning in spoken or written discourse requires students to have a functional grasp of the mathematics register. For second language students the challenge here can be formidable. Linguistic problems for LEP students fall into three major categories:

1. <u>Vocabulary</u>: knowledge of vocabulary is the most important aspect of second language (L2) competence when learning academic content through that language. Further, information processing in mathematics is closely tied to specialized vocabulary in terms of both specific terms unique to mathematics and lexical items which are marked with unique meanings for the mathematics register
2. <u>Syntax</u>: two major sources of difficulty for LEP students include: (a) the lack of one-to-one correspondence between conventional symbols and their verbal expression, and (b) the extensive use of logical connectors linking prepositions.
3. <u>Discourse (text)</u>: the ability to solve word problems with its requirements of understanding various dimensions of meaning, linguistic and mathematical, and planning solution strategies draws on both cognitive and metacognitive functioning.

Kessler et al also note that the acquisition of any second language is a sociocognitive process resulting from efforts to participate in the communicative interactions. As such, mathematics can provide a context in which second language acquisition can occur. The challenge is to both stimulate second language acquisition and to develop mathematical reasoning and practice. The extent to which each second language student can make meaning and use of the mathematical concepts presented and engage in active expression of the second language in the process of instruction will determine the success of the student in learning mathematics. Literacy is crucial in the development of mathematical reasoning. For LEP students lacking reading and writing skills, experience in writing and solving their own mathematical problems can contribute to successful language acquisition and mathematical performance.

Dale and Cuevas (1992) discuss integrating mathematics and language learning in a chapter of *The Multicultural Classroom: Readings for Content-Area Teachers* by Richard-Amato and Snow. In this chapter the authors seek to provide content area teachers with a working knowledge of the challenge of teaching second language learners mathematics and the strategies and techniques to make that teaching effective.

A beginning point for effective teaching is a realization that mathematics is a language and has a "register", that is, a subset of language composed of meanings appropriate to the communication of mathematical ideas together with the terms or vocabulary used in expressing these ideas and the structures or sentences in which these terms appear. So then, mathematics registers include unique vocabulary, syntax (sentence structure), semantic properties (truth conditions), and discourse (text) features.

- <u>Vocabulary</u> includes words that are specific to mathematics (i.e., divisor, denominator) and words from natural language (i.e., equal, rational) that take on different meanings when used in mathematics. Often complex strings of words and phrases are combined to form a new concept (i.e., least common multiple, a quarter of the apples) and there are ways in which the same mathematics operation can be signaled (i.e., add/and, plus/sum, combine/increased by). Meanings of words and terms are determined by the context in which they occur. Symbols are also used in expressing mathematical concepts and processes. Students must be able to sort out and understand the complexities and nuances of vocabulary to be successful in understanding mathematics register.
- <u>Syntax</u> includes special syntactic structures and special styles of presentation. One of the principal characteristics of the syntax used in a mathematical expression is the lack of one-to-one correspondence between mathematical symbols and the words they present. Another characteristic is its frequent use of logical connectors which are used to develop and link abstract ideas and concepts (i.e., if . . . then, if and only if, because). When students read mathematics texts, they must be able to recognize these logical connectors, the situations in which they appear, and which situation is signaled—similarity, contradiction, cause and effect or reason and result, chronological or logical sequence.
- <u>Semantics</u> includes the process of making meaning from the language, in this case, mathematical language, and students' ability to make inferences in mathematical language often depends on the language user's knowledge of how reference is indicated. To successfully complete mathematical problems students must be able to make an adequate translation from the words of the problem into the symbolic representation of the solution equation.
- <u>Discourse Features</u> refers to the "chunks" of language—sentences or groups of sentences or paragraphs—that function together as textual units, each with a specific meaning and purpose in mathematics. This includes the items listed above. To understand a unit of mathematical discourse, students must not only be proficient in mathematics language, they must also have a background in mathematics in order to construct the context needed to process cognitively complex information.

Dale and Cuevas assert that language is the vehicle of learning and instruction. Given this point of view, they continue that the role of language in learning is not always clear, although it is likely that its role is crucial in the following ways:

1. There appears to be a high correlation between reading skills and mathematics achievement, particularly when the tasks involve reading texts or solving word problems. Preliminary research

evidence appears to indicate that this correlation may be even stronger for LEP students performing tasks in their second language.
2. On a deeper level, language works as a mediator for mathematical thinking and metacognition. Whether the thinking defines the language or the language defines the thinking remains to be answered. Probably both occur. The important point is that mathematical thinking, mediated by linguistic processes, is a prerequisite for mathematics achievement.
3. LEP students who must learn mathematics through their second language must reach a minimal level of proficiency in the cognitive, academic skills required of mathematics and in the language skills used to express the mathematical skills (see Cummins' concept of CALP—Cognitive Academic Language Proficiency).
4. The language used in mathematics is intricately connected to the mathematical concepts, processes, and applications it expresses. Therefore, mathematics instruction, particularly for LEP students learning their second language, should integrate mathematics and language skills.

With these points in mind, the authors urge educators to realize that the classroom environment in which language is taught through mathematics content should be carefully structured so that second language (L2) acquisition can occur. This means that instructional activities should promote L2 development through a natural process in which the focus is not on language per se but on communicating the concepts, processes, and applications of mathematics. For this to happen, teachers must monitor students for comprehension, which requires both quality instruction and ample time for students to respond. Teachers must also, whenever possible, construct instructional activities on students' real life experiences and prior knowledge and provide situations that promote student interaction.

Dale and Cuevas provide a framework to teachers for constructing meaningful lessons that address the concerns above. SLAMS or Second Language Approach to Mathematics has two strands, one focusing on mathematics content and one emphasizing related language skills. This process is based on the assumption that in order for a student to master the mathematics concepts presented in class, the language concepts must be addressed and mastered. It is also assumed that by teaching the language together with the content, understanding of the material will be facilitated. The Content Strand encompasses strategies for analysis and diagnosis of mathematics skills, followed by preventative or prescriptive activities. The components of this strand include: analysis of concept/skills, diagnosis, preventative strategies, and the lesson. The Language Strand follows the same path. The components of this strand include: analysis of the language used, diagnosis of language skills, preventative language strategies, and the lesson.

Henderson and Landesman (1992) reported the effects of thematically integrated mathematics instruction on achievement, attitudes, and motivation in mathematics among middle school students of Mexican descent. Thematic instruction, it is posited in this study, may provide an effective way to contextualize instruction. It incorporates a concrete learning-by-doing orientation and has the potential to facilitate cooperative and interactive learning opportunities in the classroom. Similarly, cooperative learning can provide opportunities for hands-on activities that result in products on which students perform mental operations, and in situations that engage students in the use of concepts and materials. These features have been identified as characteristics of classrooms that have proved effective for Hispanic students with limited English proficiency.

Since it has been held that there is a substantial relationship between what schools actually teach and what students learn, it holds that fragmented and decontextualized mathematics instruction poses barriers for understanding and achievement, while a thematic, contextualized mathematics instruction provides students with opportunities to make sense of mathematics concepts and problem-solving strategies.

This study utilized a middle school serving a population of predominately Mexican descent, with 90% of the students being Hispanic and 66% being identified as limited English speakers. Seventh grade students were divided into classes that were thematically taught and traditionally taught. Theme students were grouped into two heterogeneous classes, one of which was taught in English only, while the other was taught bilingually. The second year students were not randomly assigned to treatment groups.

The results of the study found that in both the theme and comparison conditions students demonstrated significant gains in computation, but neither group exceeded the other. However, rather than a negative aspect, the results show that thematically taught students achieve as well as traditionally taught students. The difference in essence is the quality of the learning displayed by students: the authors reasoned that computation would receive a great deal of attention in comparison classes, and theme students would be exercising computational skills in meaningful problem-solving contexts rather than practicing skills and algorithms as isolated ends in themselves. As well as making gains in achievement levels, students also expressed a high degree of liking for mathematics, students considered themselves to be good at mathematics, and the majority indicated they wanted to take more mathematics.

Mather and Chiodo (1994) discussed teaching mathematics to elementary ELL students. They noted that it is important to evaluate the methods teachers are using to teach these students in general and to evaluate the methods used in the process of teaching mathematics in particular. To decide which methods to use, the authors question whether or not an understanding of English is needed to understand and acquire mathematics skills. To illustrate that the answer to this question needs careful consideration, the authors cite several situations. First, numerals are not universally interpreted the same, particularly with regard to notation (i.e., where the comma goes in large numbers and what it means to a given culture). Second, one's culture can interfere with the learning of mathematical concepts (e.g., one Native American culture does not have a concept for line; the Hmong culture does not have a concept for fractions, etc.). Third, the authors assert that research has shown that language skills (i.e., reading and comprehension) outpaced the grade level of mathematics instruction. Fourth, the authors assert that mathematics is deeply affected by the components of language—that is, vocabulary, syntax, semantics, and discourse.

Given these points, Mather and Chiodo believe that special teaching strategies, methods, or both should be used when teaching mathematics to LEP students. To this end, the authors provide this list of strategies:

1. Stress understanding rather than rote computational procedures.
2. Provide profound exposure to manipulative, concrete, sensory, and hands-on activities, not to replace discussion, but to support it.
3. Use cooperative learning (small group activities) and minimize individual seatwork.
4. Provide opportunities for peer tutoring—preferably by another LEP or bilingual student who understands the concepts.
5. Include guided practice with close monitoring of students.
6. Use reinforcement, reward, and total motivational systems.
7. Emphasize multicultural referents and relevancy in lessons.
8. Use second language texts, materials, and resources as much as possible.
9. Use limited, simplified instruction (using caution to retain the essence of the original content and problems); limited use of pronouns and adjectives; instruction with pauses and repetition; concerted efforts to be aware of and to explain any culturally-based terms.
10. Use basic mathematics vocabulary in the second language for individualized instruction whenever possible. Note that vocabulary should not be used to focus on key words but should be used in context to develop understanding.
11. Model expected student behavior.
12. Be aware of how other countries and cultures teach basic mathematical concepts.
13. Use programs that are designed to increase hands-on activities and relevancy and minimize abstraction.
14. Use such methods as Direct Instruction (e.g., Active Mathematics Teaching, or Cognitively Guided Instruction.

Anstrom (1998) in preparing a report for the US Department of Education researched the instruction of secondary-level language minority students in mainstream courses by looking at practices and standards. The author's intent was to provide teachers and teacher educators insight into how mainstream classroom instruction can be designed and implemented to enhance the academic achievement of language minority students. The author's findings are reported to be found based in current research.

Regarding the core subject of mathematics, Anstrom states that teachers are called to act as facilitators and to provide intellectually challenging problems that encourage students to develop mathematically. Specifically, the author identified these classroom strategies.

1. Emphasize problem-solving and communication
2. Relate mathematics to students' real life experiences
3. Encourage mathematical communication
4. Put students' needs and interests first
5. Orchestrate classroom discourse
6. Teach the language of mathematics
7. Create classroom environments rich in language and content
8. Encourage exploration and reflection through journal writing
9. Integrate reading into mathematics instructions
10. Make connections to students' prior knowledge and experience
11. Vary instructional methods

Moschkovich (2002) explored three perspectives on bilingual mathematics learners and considered how a situated and sociocultural perspective can inform work in this area. She asserts that research needs to address the relation between language and mathematics learning from a perspective that combines current perspectives of mathematics learning with current perspectives of language, bilingualism, and classroom discourse. The author notes that mathematics and ELL research has progressed from looking at the problems bilingual Latino students have faced in solving word problems, understanding vocabulary, or translating from English to mathematical symbols to how students construct knowledge, negotiate meanings, and participate in mathematical communication. For students now in reform oriented mathematics classrooms, students are expected to communicate mathematically in written and oral forms and participate in mathematical practices, such as explaining solution processes, describing conjectures, proving conclusions, and presenting arguments.

To broaden her discussion, Moschlovich discussed three perspectives and the value and limits of each. First, one perspective is to say that students are acquiring vocabulary. This perspective defines learning mathematics as learning to carry out computations or solve traditional words problems, and emphasizes acquiring vocabulary as the central issue with which second-language learners are grappling when learning mathematics. The limitations of this perspective are apparent when viewed in more current practices of learning to communicate mathematically—a focus on vocabulary becomes too narrow a perspective. A vocabulary perspective fails to go beyond simple recognition and use of a lexicon of mathematical terms and ignores the multiple meanings of words and other facets of language as well as limiting a teacher's capability to understand what a student actually knows and uses in his/her thought processes. Second, another perspective is the notion of mathematics register. Rather than focusing on a list of words, this perspective describes learning mathematics as constructing multiple meanings for words. Key in this perspective is the concept of register, which is a language variety associated with a particular situation of use. This brings complexity to bear on a view of language and the learning of mathematics. When considering the idea of multiple meanings which can create obstacles in mathematical conversations because the teacher must not only navigate and understand student's language and personal meanings, but also the register of mathematics and its variations in classroom use. Students must learn mathematics and shift from everyday to more mathematical and precise meanings. When considering this perspective, the author warns that register is more than a list of technical worlds and phrases, that utterances are situationally located in a context, and that everyday language can also provide resources for mathematical communication. Third, the final perspective is a situated sociocultural perspective, which can be used as a description of resources and competencies and widens what counts as competence in mathematical communication. Several implications are located in this perspective: learning is a discursive activity; participating in community of practice; using multiple material, linguistic, and social resources; and that participants bring multiple views to a situation; that representations have multiple meanings for participants; and that these multiple meanings are negotiated through conversations. Through these activities the teacher has more information to lead the learning experience and to evaluate students' understanding and students have more opportunities to participate in valued mathematical discourse practices, such as being precise or using representations to support claims. The author provides two classroom examples to support her contention.

Qualitative Studies on MELL

This section presents two qualitative studies on mathematics and ELL students. Qualitative studies typically describe situations or programs and from those detailed works seek to provide findings that can be meaningful or provide starting points for further research. The first study by Hewlett-Gomez and Solis describes a program for teaching middle school ELL students in a Dual Language setting. In the second study Reyes and Fletcher looked at six highly successful school districts to identify major themes that could describe successful ways of working with migrant students.

Hewlett-Gomez and Solis (1995) reported on the Literacy Program for Recent Immigrant Students, which is a English/Spanish program of instruction for recently immigrated secondary students located in a south Texas school district. This program was utilized in two middle school campuses serving grades 6, 7, and 8. The program used a team teaching approach—a certified and endorsed bilingual/ESL teacher who taught in English and a Spanish-speaking *professora*—a teacher from Mexico hired as an assistant—who taught in Spanish.

Five features of the program are noteworthy the authors state because most secondary programs lack them:

1. a sensitivity to students with the most limited English skills and mainstream experiences,
2. instruction in two languages,
3. comprehensive instruction which extends horizontally to the four literacy skills—listening, speaking, reading and writing, and vertically to address all language proficiency levels,
4. both language and content instruction, and
5. the incorporation of student's cultural experiences into the curriculum

The program featured five components, including (a) identification, assessment, and placement; (b) curriculum, instruction, and materials; (c) staffing; (d) staff development; and (e) parental involvement.

The program begins with a comprehensive assessment of students, including formal assessment, a collective, informal observation procedure, and analysis of student characteristics for language ability. Based on this assessment procedure, students were divided into Dimension I and Dimension II groupings. Dimension I students had little or no Spanish academic skills, while Dimension II students possessed varying levels of Spanish academic skills which were deemed sufficient to benefit from a more accelerated instruction in Spanish. Dual language is provided for Dimension I and II students with a three year instructional plan to help each group develop skills at their proficiency levels. This instructional plan includes (a) an integrated, disciplinary curriculum with a whole language philosophy, (b) instructional strategies and materials with specialized instruction in both Spanish and English in reading, language arts, and writing, with a content-based model for mathematics, science, and social studies, and (c) a culture component integrated across the curriculum. Students were taught by a team of two teachers—one who taught in English and one who taught in Spanish. The Spanish duties were provided by *professoras*—Mexican licensed teachers with credentials and degrees from Mexican teacher preparation schools. Staff development was extensive for both members of the teaching team. Specifically, the training included knowledge on (a) the natural approach and second language acquisition, (b) whole language philosophy, (c) CALLA, (d) the Counseling Language Learning Method, (e) transitional reading techniques, (f) process writing, (g) adaptations of key approaches for Spanish instruction, (h) cooperative learning and grouping patterns, (i) classroom management for lesson planning and positive behavior modifications, (j) instructional lesson planning, (k) strategies to integrate culture into the classroom, and (l) informal assessment techniques. Parental involvement linked parent concerns and their needs for helping their children in school. Parental training opportunities and the use of a parental liaison were utilized to link the school to the home.

The program was evaluated following the first year of implementation. It was found to provide support for the program design's success. In brief, test assessment results showed some quantitative gains in Spanish and English achievement. Comments by teachers, students, and parents about the program indicated that the program was well received by students, teachers, and administrators. In particular, students felt like they were learning in school, wanted to attend classes, and were liked by their peers.

Reyes and Fletcher (2003), citing the lack of literature on effective methods for teaching mathematics to migrant students, conducted a qualitative study of six highly successful school districts (four in Texas, one in Illinois, and one in Montana) who had met with success in educating migrant students. These campuses met the standards set for their criteria of at least 70 percent of test-taking migrants students who passed all areas (math, reading, and writing) of the Texas Assessment of Academic Skills (TAAS) state mandated test.

Reyes and Fletcher found four major themes emerged from their interviews and observations regarding math programs and how they worked with migrant students: (1) a workplace culture focused on instructional improvement, (2) respect for all students, (3) student centered instruction, and (4) a spiraling curriculum that emphasized constant review. Each of the themes was expanded and the accompanying elaborations provide insight into the importance of the studies findings. The themes and expansions are listed below.

1. Workplace culture focused on instructional improvement
    a. Extensive collaboration and communication among teachers
    b. A pervasive sense of collegiality
    c. A shared vision with a clearly articulated set of objectives, and scope and sequence for reaching that vision
    d. Continuous reflection regarding specific curriculum and pedagogical techniques appropriate for achieving targeted objectives
2. Respect for all students
    a. The belief that all students can learn
    b. The belief in the student's ability to perform, which results in high expectations
    c. A respect for the culture of migrancy
    d. The use of the culture of migrancy to teach
    e. Teachers who really care
3. Student centered instruction
    a. An emphasis on choosing relevant content to teach skills and concepts
    b. Teaching problem solving as a life skill, especially in the elementary schools
    c. Student collaboration in the learning process
    d. Providing support systems for students
4. Spiraling curriculum that emphasizes constant review
    a. This process of constant review (1) is an integral part of the scope and sequence for most math courses, (2) requires making connections between concepts, and (3) maintains high expectations from top to bottom

In the discussion of the studies findings, the author assert that the high degree of success the study schools have had with migrant students indicate that the beyond the similarities shared by the schools in the form of the themes above, there are two additional lessons to be learned. First, organizational culture appears to exert more influence on the success these schools have with migrant students than specific curricula or pedagogical techniques. Second, state standardized testing in the form of the TAAS has provided an impetus for these districts to focus their priorities.

Quantitative Studies Related to MELL

The section reports on qualitative studies related to mathematics and ELL students generally. Studies selected for this section contained either a study performed in an educational context, featured a meta-analysis of existing research, or reported on a longitudinal study. Not all studies were exclusive to mathematics and ELL; however, several studies on broader topics, which include ELL students were added to complete the picture of work with ELL students. Two studies conducted by Bernardo looked at bilingual students abilities to decipher word problems. Leon also examined students' abilities to decode word problems, but the sample focused on Hispanic students classified as learning disabled. Wang and Goldschmidt studied middle school students to determine if language proficiency and immigrant status affects student choices of courses and mathematics achievement. Stevens et al conducted a study which looked at personal qualities held by students and mathematical achievement in a comparison of white and Hispanic high school students. Greene performed a meta-analysis of bilingual programs to determine the efficacy of teaching bilingual students in their native language. Baker et al conducted a meta-analysis of low achieving students in mathematics. Nye et al reported on a Tennessee longitudinal study that examined the effects of class size on mathematics achievement. Finally, Thomas and Collier conducted a national study examining the efficacy of language instruction programs and their success in serving language minority students.

Leon (1994) reported on a study which examined the effects of extraneous information on mathematical word problems on the performance of Hispanics classified as learning disabled children. This study was conducted in two school districts in the Western New York area. These school districts were similar in that both districts have a large Spanish-speaking community, and they provide bilingual special education services to culturally and linguistically diverse exceptional children. The sample consisted of forty-one (N=41) Hispanic learning disabled students. According to the cumulative records, these students were receiving instructional services in self-contained and resource classrooms. The subjects ranged in ages 9 to 14 years. To set up the study, two sets of mathematical word problems were developed incorporating addition and subtraction with extraneous wording and without extraneous wording. Set A consisted of twenty-four (24) mathematical word problems in English. Set B which was the Spanish version of Set A, also consisted of twenty-four (24) mathematical word problems. Each mathematical word problem contained four sentence statements. Problems were constructed as single-step with responses that totaled less than ten (10). The vocabulary used approximated a primary grade level and was culturally appropriate. Mathematical word problems with extraneous information did not contain a quantitative characteristic, but in order to keep all the problems with four sentence statements, a sentence relevant to the information in the problem was included.

Leon (1994) in discussing the results of the study suggest that students appeared to experience difficulties in discriminating between essential from non-essential information in order to arrive at a correct solution. Thus, based of the responses given, the majority of the students knew how to execute the arithmetic computations, but they did not know how to apply them to mathematical word problem-solving solutions. In addition Leon proposes that the results of this study are relevant to the fields of bilingual special education and mathematics education. From an instructional point of view, these results have important implications in the educational programming of Hispanic learning disabled children at all grade and age levels. The building of mathematics concepts and vocabulary is crucial; therefore, both should be somehow related to the experiences the child brings into the classroom. This approach may facilitate the thinking process in order to get a correct answer.

Greene (1998) performed a meta-analysis on the effectiveness of bilingual education and found that despite a limited number of studies that met a strict selection process (quality of research design) children with limited English proficiency who are taught using at least some of their native language perform significantly better on standardized tests than similar children who are taught only in English. The limited number of studies for review makes it difficult, according to the author, to address other important issues, such as the ideal length of time students should be in bilingual programs, the ideal amount of native language that should be used in instruction, and the age groups in which these techniques are most appropriate. The author notes cautiously, “It is possible that the individual needs of students are so varied that there may be no simple set of ideal policies.”

Wang and Goldschmidt (1999) conducted a study in which they examined data from a larger urban school district in California to investigate the roles of opportunity to learn (OTL), language proficiency, and immigrant status on 2,443 middle school students' mathematics achievement over three years. This study concentrated on the specific issue of curriculum equity in terms of coursework measures and investigated the relationship between students' mathematics achievement and their opportunity to learn in terms of course taking. The study had two objectives. The first objective was to examine whether students' language proficiency and immigrant status affect students' course taking and to what extent, and whether course taking directly and indirectly affects students' mathematics achievement and growth. The second objective was to investigate how students' course taking affects their mathematics achievement and growth rates after controlling for their previous achievement, language proficiency, immigrant status, and other background values. The authors examined whether there were significant differences in students' course taking by ethnicity, language, and immigrant status.

According to the authors, five important results emerged that partially account for differences in eighth grade mathematics achievement.

- <u>First, when the type of mathematics course taken was equalized, girls' mathematics performance was statistically lower than the boys' performance</u>. The results indicate that the gender gap is not simply a result of girls not pursuing more mathematics courses in high school. Unequal treatment and differences in OTL in middle schools may negatively affect girls' attitudes towards mathematics. Girls also may avoid pursuing advanced mathematics courses in high school, at which point the gender gap may become a function of both inequitable treatment and differences in course taking.
- <u>Second, differences in SES explain some achievement differences, but are much less important that other student level characteristics</u>. Students who received free lunch scored 8 points lower than did students ineligible for free lunch.
- <u>Third, students' course taking patterns vary across their language proficiency, immigrant status, and ethnicity</u>. The middle school students' course-taking distribution by language proficiency is markedly different for non-immigrants and immigrants, especially for re-designated students. Other research has shown that African American and Hispanics students are usually underrepresented in algebra and honors courses.
- <u>Fourth, students' course taking explains their mathematics achievement differences even after considering students' descriptive characteristics, language proficiency, immigrant status, and SES</u>. Students who study algebra, honors mathematics, or an elective mathematics course had significantly higher test scores compared with students enrolled in standard mathematics classes. Students who took a minimum standard course performed significantly lower than did students in a standard mathematics class. Also, students in ESL mathematics course had low test scores, even though the difference was not statistically significant. Students who took different courses also had different growth rates. Students who took elective mathematics had the highest growth rates and students in minimum standard classes had the slowest growth rates.
- <u>Fifth, the results indicate that there is an important interplay between students' OTL and language proficiency</u>. The authors, results reveal that although re-designated students had a sufficient grasp of English to be classified as being fully English proficient, their level of English proficiency was not enough for them to succeed in advanced courses. Those students required some language support after re-designation. On the other hand, ESL mathematics is as beneficial for LEP students as standard mathematics is for English only students. This finding should disapprove the belief that non-native English speakers are inferior in learning content areas because they do not speak English.

Baker, Gersten, and Lee (2001) conducted a meta-analysis of eighteen studies of the performance of low achieving students in mathematics. These authors assert that the body of research reviewed suggests that there are two areas where findings converge. The first is that providing precise information on student progress and specific areas of students' strengths and weaknesses in mathematics can enhance achievement for this population. The second is that having students work with partners in a structured format as they work out mathematics problems is likely to be an effective means to enhance achievement. Verbalizing their ideas may well enhance their understanding of mathematics. The study also suggests that providing parents and family members of low achieving students with specific feedback on how well their children are doing, and encouraging them to reward their efforts and accomplishments is a positive to a solid mathematics program. When examining core issues of curriculum and instruction for low achieving students the research base is at best, fragmentary. Even so, the authors found some suggestion that clear explicit instruction with adequate practice and a wide range of example is a good means to build foundational skills and knowledge of complex topics such as fractions, rations, decimals, and equivalence. Similarly the research suggests that, after initial instruction, learning can be enhanced by having students work through complex real world problems that involve these concepts. Finally, there is some evidence that explicit practice and rehearsal on generic problem solving strategies is helpful but too limited to suggest clear recommendations of practice.

Nye, Hedges, & Konstantopoulos (2001) reported on a longitudinal study conducted in Tennessee in which the results verified that small class size has immediate positive effects on academic achievement (in this case, mathematics). Unlike most other early education interventions, these effects persisted for several years after the children returned to regular-sized classes. In this report, student achievement in mathematics was examined at the ninth grade, a full six years after the initial samples were studied.

In the original study the experiment randomly assigned kindergarten students to small classes (with 13-17 students), larger classes (with 22-26 students), or larger classes with a full-time classroom aide. Teachers were also randomly assigned to classes of different types. These assignments of students and teachers to class types were maintained through the third grade. Some students entered the study in the first grade or subsequent grades and were randomly assigned to classes at that time.

For the present report, the researchers looked at the effects of the original study on class size for ninth grade students' ability to perform in mathematics. The results of the study show the effect of small classes was significant and positive. The minority x small class size analysis was significant and indicated that the small class effect was larger for minority students than white students. This was true for students who had at least one or more years in the study sample and who were assigned to a small class size in Grade 3 and for students who had four years in the study sample. As the authors note, "lasting effects were found for all kinds of students in all kinds of schools, but the benefits of small classes were significantly larger for minority students. Thus, small classes may be a way to benefit all students while reducing the gap in achievement between White and minority students."

Bernardo (2002) conducted a study to determine whether Filipino-English bilingual students' understanding and solving of word problems in arithmetic differed when the problems were in the students' first and second languages. Participants in the study were ninety-two Grade 2 students from nine private elementary schools in metropolitan Manila who had just been introduced to word problem solving and were thus novices at the task. Students were enrolled in bilingual schools in which some subjects, including mathematics, were taught in English, using English language textbooks, materials, and assessment procedures, and other subjects are taught in Filipino, using Filipino language textbooks, materials, and assessment procedures. The author based problems used in the study on 18 story problems used by Riley and Greeno (1998), which consisted of six specific problem types within each of the three major problems types:

1. Combine problems: A subset or superset must be computed by using information given about two other sets;
2. Change problems: A starting set is changed by transferring items in or out, and the cardinality of the starting set, transfer set, or the results must be computed by using information given about the two other sets; and
3. Compare problems: The cardinality of one set must be computed by comparing information given about the sets.

Students were tested individually by a research assistant during school hours in the library or in the classroom. Students were asked which language they preferred to use in the session and at home. Testing began with problems read aloud twice. All responses were coded as correct or incorrect.

The results of the study revealed that having either Filipino or English as a first language did not effect an ability to understand word problem texts in arithmetic. The results also revealed that there was a significant interaction between a student's first language and the language of the problem. Students with Filipino as a first language got more problems in Filipino correct than in English and students with English as a first language got more problems in English correct than in Filipino. Further, the results showed that the arithmetic problem solving of bilingual students was poorer with word problems in both the first and second languages compared with the same problems presented in a purely numerical form. "The results show that at least for word problem solving, a task that has a clear linguistic component, the application of mathematical knowledge and skills is affected, possibly even constrained, by whether the student is able to effectively undertake the requisite linguistic processes. Moreover, the results of the study show that for bilingual students, all these processes seem to be more effective when the problem to be solved is presented in the first language rather than the second language."

Thomas and Collier (2002) conducted a national study built on fourteen years of related research on language minority students and their academic achievement. The study, *National Study of School Effectiveness for Language Minority Students' Long Term Academic Achievement*, collected data from five school districts throughout the US and attempted to understand how effective were various programs serving language minority students. Four distinct theoretical program designs were included:

1. Two-Way Bilingual Immersion programs: these promote academic achievement, bilingualism, and biliteracy for ELLs and native English speakers and typically last for five to six years,
2. One-Way Developmental Bilingual Education programs: offer instruction only to language minority students of one language background (including ELLs) and typically last for five to six years,
3. Transitional Bilingual Education programs offer classes presented in the ELLs native language for two to three years and then receive all English instruction, and
4. English as a Second Language programs for ELLs: teach English to ELLs through academic content areas.

The sample for this study included 210,054 student records and followed student achievement for a period of five years by looking at and comparing the performance of ELLs and non-ELLs. (It is well documented that there is a gap between ELLs and non-ELLs in academic achievement.) The authors examined results from standardized tests (reading, language arts, and math) and variables such as socioeconomic status, number of years of primary language schooling, and gender differences for influence on academic achievement.

The results of the study include the following:

- 90/10 and 50/50 Two-Way Bilingual Immersion and One-Way Developmental Bilingual Education programs are the only programs found to date that assist students to fully reach the 50th percentile in both their native language and English in all subject areas and to maintain that high level of achievement, ore reach even higher levels through the end of their schooling. The fewest dropouts came from these programs.
    - Two-Way Developmental Bilingual Education including Content ESL: 61 NCE (Normal Curve Equivalents); shows improvement until Grade 11
    - One-Way Developmental Bilingual Education including Content ESL: 52 NCE; shows improvement until Grade 11
    - Transitional Bilingual Education including Content ESL: 40 NCE; shows early improvements to about Grade 3 and then levels off to Grade 11
    - Transitional Bilingual Education + ESL with both taught traditionally: 35 NCE; shows improvement to about Grade 3 and then declines slightly to Grade 11
    - ESL taught through Academic Content (no Language 1): 34 NCE; shows improvement to about Grade 3 and then declines slightly to Grade 11
    - ESL Pullout (no Language 1) taught traditionally: 24 NCE; shows improvement to approximately Grade 3 then a steady decline until Grade 11
- ELLs who attended only English mainstream programs because their parents refused language support services showed large decreases in reading and math achievement by Grade 5 when compared to students who participated in language support programs. The largest number of dropouts came from this group.
- When ELLs initially exit a language support program into the English mainstream, those schooled in all-English medium programs (ESL) outperform those schooled in the bilingual programs when

tested in English. The students schooled in bilingual programs, however, reach the same levels of achievement as those schooled all in English by the middle school years. Further, during the high school years, the students schooled in bilingual programs outperformed the students schooled in all English.

- The amount of formal primary language schooling that a student has received is the strongest predictor of second language achievement. That is, the greater the number of years of primary language, grade-level schooling a student has received, the higher his/her English achievement will be.

Based on these findings, the authors propose that in order to close the average achievement gap between ELLs and native English speakers (non-ELLs), language support programs must be well implemented, not segregated, sustained for five to six years, and demonstrate achievement gains of more than the average yearly progress of the non-ELL group each year until the gap is closed. The problem here is that the achievement gap is at best a moving target since non-ELLs progress academically each year for their grade level, while ELLs typically fall further behind with each grade level. Thus, even the most effective language support programs can only close half of the achievement gap in two to three years.

Stevens, Olivarez, Lan, & Tallent-Runnel (2004) conducted a study to evaluate a theoretical model that describes relationships involving personal qualities, including self-efficacy and motivational orientation, and variables associated with mathematical achievement, such as performance and the intent to take additional mathematics courses, across Hispanic and Caucasian students. The resulting path model tested the relations among six correlated factors: (a) ability, (b) mathematics self-efficacy, (c) motivational orientation, (d) prior mathematics achievement, (e) mathematics performance, and (f) intention to enroll in additional mathematics courses. The goal was to substantiate the relationships found between personal qualities and mathematics achievement in a Hispanic sample. Participants included 317 9th grade and 100 10th grade students; ages ranged form 14 to 17 years of age. Fifty-three percent (53%) of the students described themselves as being Hispanic, and 30% identified themselves as Caucasian. African American students comprised 4.6% of this sample; 3.6% of the remaining descriptions did not fit into the aforementioned groups. Thirty-nine students (9.4%) did not report their ethnicity. Although all students were included in the total sample analyses, only the reports of Caucasian and Hispanic students were evaluated in group analyses. That decision reflected the purpose of this study and was not an indication of a lack of importance of the mathematical skills of students who belonged to other ethnic groups. The results of the study revealed several things. First, the results indicated that students' beliefs and motivation played an important role in mathematics achievement for both Caucasian and Hispanic students. Second, the results revealed that in comparison to Caucasian students, Hispanic students' mathematics performance is significantly below that of that their Caucasian peers. Third, the results suggested that Hispanic students do not view their ability when considering their self-efficacy and performance. Fourth, Hispanic students reported that their prior lack of success in mathematics achievement existed before this study and that these beliefs had a greater impact for Hispanic students than Caucasian students. Possible explanations for this phenomena include the fact that Hispanics may not have the same opportunities as Caucasian students for verbal persuasion—verbal sources of support from key people in those students' lives whether teachers, peers, or parents—and access to models—members of the Hispanic community who can communicate the importance and use of mathematics—that encourage participation in mathematics.

Bernardo and Calleja (2005) in a follow-up study examined mathematical word problems. Word problems are an integral part of mathematics education and allow students to apply their mathematical knowledge and skills to real world situations. In this study the authors explored the effects of the language of presentation on solving word problems among Filipino-English speakers. Specifically, the authors explored the effects of stating the word problems either in the first or second language of bilingual problem solvers on how they would solve word problems that required the application of real world constraints. That is, would there be different patterns of solutions between students solving problems in Filipino or English and would there be a difference if students used real life constraints in their solutions in either language.

Fifth grade students were given eighteen problems with two types of problems. The first type was a standard problem that could be solved by straightforward arithmetic operations. The second type was a version of the standard problem that was problematic because if the student considered real life facts, constraints, or knowledge, the answer to the problem could not be solved using straightforward arithmetic problem solving. Students were to complete problems either in English or Filipino. Results of the study showed that students in the sample almost never considered real life applications to solve the problem. The results also showed that the tendency to ignore realistic knowledge is strong regardless of which language the problem is presented. The results also showed that students were more successful in applying the appropriate arithmetic procedures with problems written in Filipino than in English even though mathematics was taught in English (including texts, readings, and word problems). Despite what seems to be obvious—that students perform best in their native language—the authors assert that the language used in math word problems might not always be an important factor. It could be that certain problems or difficulties that bilingual students have with solving word problems, such as the tendency to not consider or apply real world knowledge, might not be associated with language at all. These issues might rather be associated with instructional issues, such as poorly developed learning experiences that give rise to the acquisition of problem solving strategies—the lack of experience with real life situation based problems versus straightforward simplistic word problems and the lack of teacher modeled strategies for solving real world problems.

Literacy and ELL

The articles in this section present differences in discourse features between Hispanics and white students (Montano-Harmon) and the connection between literacy and mathematics instruction (Cardelle-Elawar, Raborn, Jarrett, Kotsopoulos). When connected to the literature already presented, this section makes the case for a strong connection between literacy and mathematics instruction for ELL students.

Montano-Harmon (1991) presented a study that identified differences in discourse features of compositions written in Spanish by secondary school students in Mexico with those written in English by Anglo-American students in the United States. Noting that past studies of discourse patterns in written texts in various languages indicate that the logical development of text varies depending on the native language of the writer, Montano-Harmon sought to contribute to the understanding of how native speakers of Mexican Spanish organize their expository writing and to the application of this understanding to the development of literacy skills in Spanish language programs. To accomplish this goal, Montano-Harmon analyzed the similarities and differences in discourse features, including discourse patterns, of texts written by ninth grade high school students from four linguistic groups:

1. Mexican students in Mexico who are native speakers of Spanish writing in Spanish;
2. English-as-a-Second-Language (ESL) students who are native speakers of Mexican Spanish who are students in the United States writing in English;
3. Mexican-American/Chicano students who are dominant speakers of English writing in English; and
4. Anglo-American students who are native speakers of English writing in English.

Written texts—200 total: 50 for each group—were analyzed by expert readers and linguistic data were coded for statistical analysis. Comparisons of the discourse features in the texts in five areas were made including basic information about the texts (length, number of sentences, average length of sentences), types of sentences (fragments, run-on, simple, compound, complex, compound-complex), lexical cohesion (reiteration, including same word, synonyms, semantically superordinate word, general class word; collocation; conversational markers), syntactical cohesion (reference by type, reference by position, substitution, ellipsis, conjunction), and coherence or logical relationships (topic sentence, enumerative, additive, summative, resultative, explicative, illustrative, constrastive, transitional words and phrases).

The results of the study confirmed that the four linguistic groups differed significantly from each other in their use of discourse features. In particular, the discourse features in Mexican Spanish were significantly different from those in Anglo-American English. Compositions written by Mexican students in Mexico were longer with longer sentences, repetition, flowery poetic language, and the flexible sentence structures created formal, complex presentation of ideas, which differs from the linear, deductive, enumerative compositions written by Anglo-American students. Montano-Harmon warns that "since the logical development of texts is not universal but language/culture specific, it is imperative that language teachers be aware of differences in discourse features and teach these to students developing literacy skills in a second language. Otherwise, students cannot proceed beyond the sentence level in their understanding of authentic materials."

Cardelle-Elawar (1992) describes and illustrates in an article, a metacognitive approach for teaching mathematics to bilingual students that responds to two concerns: (a) a lack of knowledge about how to use existing skills and strategies, and (b) instruction that focuses student's attention on what is the right answer rather than on the process of finding answers on their own. The proposed metacognitive approach provides bilingual students with information about how skills and strategies would be helpful in solving mathematical problems and provides teachers with specific guidelines for focusing their instructional efforts. Cardelle-Elawar reports that metacognitive strategies serve three purposes: (1) they stimulate and develop students' thinking by providing insights into their own mental processes; (2) they redirect students' activities during the act of problem solving by helping students appraise their thinking; and (3) they transform the classroom into an environment where interaction and inquisitiveness are encouraged by allowing explicit discussion between teacher and student of not only what to learn, but also how and why.

The model used to teach students how to solve mathematical problems is based on the work of Meyer (1987) and holds four processes or knowledge types is required to solve mathematical problems:

- Translation requires linguistic knowledge which allows the student to understand English sentences and certain facts;
- Integration requires the student to combine each statement into a coherent representation and to have the schematic knowledge for recognition of and approaches to problem types;
- Planning and monitoring require knowledge of strategies that focus on how to solve the problem; the plan involves breaking the problem in to components, and
- Solution execution requires the student to use procedural knowledge to apply arithmetic rules accurately and efficiently while carrying out the calculations in the solution plan.

During instructional encounters, teachers seek not only to teach but also to monitor student thinking and to reveal that thinking process to students though effective feedback. By using a direct instruction model—introductory discussion, independent work, and summarization—the knowledge a student gains is reinforced throughout the lesson. Feedback is intended to encourage students to actively use the skills and strategies they have available but do not often use.

When working with low-performing students, the author suggests utilizing these strategies:

1. Focus on students' individual behavior, particularly a student's uniqueness, strengths, and weaknesses, rather than on the students' labels (e.g., low performers, minority, etc.).
2. Create a supportive atmosphere in which errors and mistakes lead to positive feedback and direction.
3. Break tasks into small steps of learning and mastery to help provide the higher level of classroom structure needed by low performing students.
4. Maximize interaction between the teacher and the student.

Raborn (1995) discusses characteristics and linguistic factors relevant to mathematics instruction of students with learning disabilities from language-minority backgrounds. In this article, Raborn states that linguistic factors must be considered when teachers plan and present mathematics instruction. She notes that math vocabulary is precise thought not familiar to all students given multiple meanings of words (such as "odd"—as *peculiar* or as *numbers not divisible by two*) found in math instruction. Further, students might have trouble distinguishing differences and making comparisons in relationships that pertain to size, speed, space, and time. To gain an understanding of these potential difficulties, Raborn suggests teachers consider issues of language comprehension, knowledge of syntax and vocabulary, and understanding of relational terms as they apply to mathematics (from Cuevas and Beech, 1983).

As a consequence, Raborn utilize the following recommendations:

- Appraise math abilities: bilingual students may have skills hidden behind an inability to express themselves in English; cognitive abilities need to be identified regardless of language proficiency.
- Select the language of instruction: bilingual students have differing abilities in both their language of origin and their second language; depending on the school's choice of instruction—whether monolingual, bilingual, or two way—a teacher who is fluent in the language of instruction must be able to understand and model the academic language of mathematics.
- Proceed from concrete to abstract: manipulation of concrete materials supported and extended through the use of language provides the basis for moving students to higher levels of abstraction, particularly when reinforces by visual labeling.
- Use strategies to help students develop concepts: through the use of concept attainment, students are able to develop skills and understandings; students compare and contrast examples that contain the attributes of the concept (positive exemplars) and with examples that do not contain those attributes (negative exemplars).
- Use math for language development: students from language-minority backgrounds profit from learning the language of math as they develop precise vocabulary, sequence, and comprehension skills in their native language; these students need time to pose their own questions and to explore ways of answering them; it is important to give students the opportunity to talk with peers and adults so they can experiment and validate their own ideas.
- Take student strengths into consideration: students can learn to benefit from their strengths when those strengths are pointed out to them. It is also helpful for teachers to model recognition and use their own personal strengths.

Jarrett (1999) in a monograph discussing the inclusive classroom presented several teaching approaches identified in current mathematics and science education literature as effective for all students and that when linked to strategies and techniques from the field of second-language acquisition should prove fruitful in teaching English language learners. Jarrett begins her discussion by noting that language is at the core of the learning experience regardless of the subject. Further, students faced with the challenge of learning a second language best learn it when it occurs in an authentic and interactive environment. She notes also the presence of two language conventions which are displayed in the classroom setting. First, social language, learned through interaction, is highly contextual, enabling language-minority students to infer meaning and interpret visual cues and body language. Second, academic language, learned and understood in isolation, is abstract and common words can take on individualized meanings so that language learners are responsible for constructing meanings and must rely on their own understanding of the language and the concepts involved. With this in mind, Jarrett believes that mathematics and science classrooms based on inquiry and problem solving hold special promise for language-minority students. It is because scientific inquiry and mathematical problem solving are based on language based activity (such as, questioning, describing, explaining, hypothesizing, debating, clarifying, elaborating, and verifying and the like) and while this may prove demanding of language learners, the potential is present for such students to benefit by these language based interactions during the learning process. Even so, the potential is fraught with pitfalls. For example, everyday vocabulary may differ from the meanings found in mathematics; or mathematical operations may be signaled in different ways through the use of terms with multiple meanings or operational connotations; or students may fail to understand the difference in mathematical concepts in words as opposed to symbols; or students may have difficulty interpreting logical connectors (e.g., if, because, however, consequently) that signify logical relationships between part of a text.

Jarrett (1999) presented strategies currently identified in mathematics and science education reform effective for all students. By linking these strategies with principles of second language acquisition, teachers can target the needs of second language students. The strategies are as follows.

1. Thematic Instruction: by organizing concepts into thematic units, the teacher is able to provide students extended time to learn content material and to become more comfortable with the language associated with the concepts. Further, by including real world application, presenting ideas and organizing activities in the context of students' home cultures; and by encouraging students talk about their prior experiences and knowledge concerning the theme second language students can link their experiences to the structured learning of the classroom.
2. Cooperative Learning: with this strategy, students use language related to the task while conversing, collaborating, and tutoring one another. By using their second language skills in authentic discourse, students are exposed to complex language structures and have opportunities to refine their communication skills by negotiating meaning through talk.
3. Inquiry and Problem Solving: Since second language students are able to develop inquiry-based and problem solving strategies before they are language proficient, they can enhance language development as they move from concrete to more abstract content while problem solving. In doing so their linguistic skills progress in complexity as well. Problem solving and inquiry activities should be relevant to student's real life experiences and prior knowledge and should provide opportunities to write reports, explanations, descriptions, their own word problems and problem solving strategies, etc.
4. Scientific Inquiry: while culturally different students may be unfamiliar with the methodology of scientific inquiry, they can be supported through an integration of explicit instruction and exploratory learning, particularly when reinforced with small group learning situations in which an authentic context is created so that language development and content are reinforced.
5. Mathematical Problem Solving: With this strategy, teachers can introduce a discussion about the vocabulary and situational context of the problem. Through helping students to break down the problem into natural grammatical phrases, providing visual cues such as graphic representation, and asking students to rephrase the problem in their own words, teachers help students to derive meaning. This will be particularly meaningful for second language students who are often literal readers.
6. Vocabulary Development: Students learn new terminology and word meanings best when they encounter them during purposeful activities and investigations. Further, by reintroducing key words in different contexts and limiting the number of words introduced in a given lesson, teachers can help students to build their content related vocabulary.
7. Classroom Discourse: Teachers can make the language of mathematics and science more comprehensible by modifying their own speech. By using an active voice, limiting the number of new terms, paraphrasing or repeating difficult concepts, and using visual supports, teachers can facilitate students' language comprehension. Teachers may find it helpful to speak slowly, enunciate clearly, use a controlled vocabulary and simple structures, and avoid idiomatic expressions. Teachers will also want to check frequently for students' understanding by eliciting requests for clarification, posing questions of varying levels of complexity, and facilitating teacher-to-student and student-to-student interaction.
8. Affective Influences: Teachers can help second language students feel welcome in the inclusive classroom by encouraging them to express their ideas, thoughts, and experiences and by showing respect for student's current language skills. Often one method that is supportive of students is helpful is to focus on *what* is being said rather than *how* it is being said.
9. Assessment: Teachers will want to use a variety of assessment methods to provide a more complete picture of students' progress and areas of need. In addition, teachers need to monitor and assess their students' language development as well as their understanding of content knowledge.

Kotsopoulos (2004) discussed mathematics education and the need for literacy in a recent newsletter. She notes that the complexities of literacy development in content-based disciplines such as mathematics are frequently underestimated. Two interrelated complexities are present and challenge literacy initiatives in content-based disciplines. First, literacy may be uniquely defined within a discipline such as mathematics and may be at odds with conventional notions of literacy found in language arts content areas such as reading and writing. Second, content-specific language registers may function independently of more familiar academic or home language registers. From the recognition of these two complexities, practitioners must realize that literacy instruction in content-based disciplines requires explicit language instruction and pedagogical approaches that enhance literacy skills unique to that content-area, not just literacy skills in general. One such content-area—mathematics—requires the recognition of a math register, which is distinct from other content-area registers. Further, one must realize that the same language may be used for different purposes under different circumstances, such as the use of language at school and the use of the same language at home. When coupled with bilingual situations, the challenge becomes doubled.

Kotsopoulos also refers to mathematical literacy as including these points:

- To engage in problem solving skills
- To make judgments
- To engage in communication about mathematics
- To derive understanding for mathematics in real contexts
- To pose problems in a variety of settings
- To recognize relationships
- To make mathematical decisions base on these relationships

Given these skills, it becomes apparent that literacy underpins any sense-making in mathematics. The challenge then is to make sure that students are able to develop literacy skills in order to be successful in learning mathematics.

Ten Common Fallacies about Bilingual Education

Crawford (1998) in an ERIC Digest report asserts that researchers have made considerable advances in the fields of psycholinguistics, second language acquisition, bilingual pedagogy, and multicultural education. Further he says that we know a great deal more about the challenges faced by English language learners and about promising strategies for overcoming them. One such strategy, bilingual education, has been the subject of increasing controversy. Although a growing body of research points to the potential benefits, there are a number of commonly held beliefs about bilingual education that run counter to research findings. Ten fallacies are presented.

Fallacy 1: English is losing ground to other languages in the United States.

More world languages are spoken in the United States today than ever before. However, this is a quantitative, not a qualitative change from earlier periods.

Fallacy 2: Newcomers to the United States are learning English more slowly now than in previous generations.

To the contrary, today's immigrants appear to be acquiring English more rapidly than ever before. While the number of minority-language speakers is projected to grow well into the next century, the number of bilinguals fluent in both English and another language is growing even faster. Between 1980 and 1990, the number of immigrants who spoke non-English languages at home increased by 59%, while the portion of this population that spoke English very well rose by 93% (Waggoner, 1995). In 1990, only 3% of U.S. residents reported speaking English less than *well* or *very well*. Only eight tenths of one percent spoke no English at all. About three in four Hispanic immigrants, after 15 years in this country, speak English on a daily basis, while 70% of their children become dominant or monolingual in English (Veltman, 1988).

Fallacy 3: The best way to learn a language is through "total immersion."

There is no credible evidence to support the "time on task" theory of language learning the claim that the more children are exposed to English, the more English they will learn. Research shows that what counts is not just the quantity, but the quality of exposure. Second-language input must be comprehensible to promote second-language acquisition (Krashen, 1996). If students are left to sink or swim in mainstream classrooms, with little or no help in understanding their lessons, they won't learn much English. If native-language instruction is used to make lessons meaningful, they will learn more English and more subject matter, too.

Fallacy 4: Children learning English are retained too long in bilingual classrooms, at the expense of English acquisition.

Time spent learning in well designed bilingual programs is learning time well spent. Knowledge and skills acquired in the native language literacy in particular are "transferable" to the second language. They do not need to be relearned in English (Krashen, 1996; Cummins, 1992). Thus, there is no reason to rush limited-English-proficient (LEP) students into the mainstream before they are ready.

Research over the past two decades has determined that, despite appearances, it takes children a long time to attain full proficiency in a second language. Often, they are quick to learn the conversational English used on the playground, but normally they need several years to acquire the cognitively demanding, decontextualized language used for academic pursuits (Collier & Thomas, 1989).

Fallacy 5: School districts provide bilingual instruction in scores of native languages.

Where children speak a number of different languages, rarely are there sufficient numbers of each language group to make bilingual instruction practical for everyone. In any case, the shortage of qualified teachers usually makes it impossible. For example, in 1994 California enrolled recently arrived immigrants from 136 different countries, but bilingual teachers were certified in only 17 languages, 96% of them in Spanish (CDE, 1995).

Fallacy 6: Bilingual education means instruction mainly in students native languages, with little instruction in English.

Before 1994, the vast majority of U.S. bilingual education programs were designed to encourage an early exit to mainstream English language classrooms, while only a tiny fraction of programs were designed to maintain the native tongues of students.

Today, a majority of bilingual programs continue to deliver a substantial portion of the curriculum in English. According to one study, school districts reported that 28% of LEP elementary school students receive no native-language instruction. Among those who do, about a third receive more than 75% of their instruction in English; a third receive from 40 to 75% in English; and one third of these receive less than 40% in English. Secondary school students are less likely to be instructed in their native language than elementary school students (Hopstock et al. 1993).

Fallacy 7: Bilingual education is far more costly than English language instruction.

All programs serving LEP students regardless of the language of instruction require additional staff training, instructional materials, and administration. So they all cost a little more than regular programs for native English speakers. But in most cases the differential is modest. A study commissioned by the California legislature examined a variety of well implemented program models and found no budgetary advantage for English-only approaches. The incremental cost was about the same each year ($175-$214) for bilingual and English immersion programs, as compared with $1,198 for English as a second language (ESL) "pullout" programs. The reason was simple: the pullout approach requires supplemental teachers, whereas in-class approaches do not (Chambers & Parrish, 1992). Nevertheless, ESL pullout remains the method of choice for many school districts, especially where LEP students are diverse, bilingual teachers are in short supply, or expertise is lacking in bilingual methodologies.

Fallacy 8: Disproportionate dropout rates for Hispanic students demonstrate the failure of bilingual education.

Hispanic dropout rates remain unacceptably high. Research has identified multiple factors associated with this problem, including recent arrival in the United States, family poverty, limited English proficiency, low academic achievement, and being retained in grade (Lockwood, 1996). No credible studies, however, have identified bilingual education among the risk factors, because bilingual programs touch only a small minority of Hispanic children.

Fallacy 9: Research is inconclusive on the benefits of bilingual education.

Some critics argue that the great majority of bilingual program evaluations are so egregiously flawed that their findings are useless. After reviewing 300 such studies, Rossell and Baker (1996) judged only 72 to be methodologically acceptable. Of these, they determined that a mere 22% supported the superiority of transitional programs over English-only instruction in reading, 9% in math, and 7% in language. Moreover, they concluded that "TBE [transitional bilingual education] is never better than structured immersion" in English. In other words, they could find little evidence that bilingual education works.

Close analysis of Rossell and Baker's claims reveals some serious flaws of their own. Krashen (1996) questions the rigor of several studies the reviewers included as methodologically acceptable all unfavorable to bilingual education and many unpublished in the professional literature. Moreover, Rossell and Baker relied heavily on program evaluations from the 1970s, when bilingual pedagogies were considerably less well developed.

Most important, Krashen (1996) shows that Rossell and Baker are content to compare programs by the labels they have been given, with little consideration of the actual pedagogies being used. They treat as equivalent all approaches called TBE, even though few program details are available in many of the studies under review. Even when program descriptions are available, Rossell and Baker sometimes ignore them.

Fallacy 10: Language-minority parents do not support bilingual education because they feel it is more important for their children to learn English than to maintain the native language.

Naturally, when pollsters place these goals in opposition, immigrant parents will opt for English by wide margins. Truly bilingual programs seek to cultivate proficiency in both tongues, and research has shown that students' native language can be maintained and developed at no cost to English. When polled on the principles underlying bilingual education for example, that developing literacy in the first language facilitates literacy development in English or that bilingualism offers cognitive and career-related advantages for the majority of parents are strongly in favor of such approaches (Krashen, 1996).

Programs Identified as Successful in Teaching ELL

This section describes three offerings that were highlighted in the literature as being successful in working with ELL students. The first two offerings were identified in the Thomas and Collier study as the two programs that are the most successful in teaching ELL students. The third is an instructional strategy based on small group instruction which features questioning and discourse as an effective mode of teaching ELL students. This section is included to provide a greater understanding of the context of ELL instruction as presented in previous sections.

<u>Dual Language Programs</u>

Torres-Guzman (2002) writing in *Directions in Language and Education* describes dual language programs and specifically attempts to delineate salient features of dual language programs for educators to keep in mind when making program and policy decisions, and to highlight the educational results ascribed to dual language programs in light of the broader debate surrounding bilingual education.

Dual language refers to an enriched bilingual/multicultural education program in which language equity is structurally defined as equal time exposure to two languages (i.e., typically the 50/50 model though some programs begin with a heavier emphasis on the native language as in the 90/10 model and then switch to the 50/50 model). Dual language programs are developmental—psychological, social, and cognitive developmental issues are taken into account—and enrichment oriented—a second language is added to the one students already know. These programs can be two-way or one-way; the primary distinction being found in the student populations they serve. Two-way programs include language minority and language majority students. One-way programs include language minority students only.

Dual language programs foster the goals of academic achievement in English and another language, development of bilingual/biliterate skills, and positive, cross-cultural attitudes. The major theoretical principals that undergird the academic and language goals are embedded in the relationship between language, learning, and cognition. The key features of all dual language programs are as follows:

Linguistic Features	Strict language separation
	Equality in language distribution
	Avoidance of simultaneous translation
	Language taught through content
	Whole language instruction
	Goals of bilingualism and biliteracy
	Heterogeneous language grouping
Sociocultural Features	Appreciation of cultural diversity
	Culturally relevant teaching
	Development of self-esteem
	Mix of language minority with English-speaking & mainstream students
	Cooperative group learning structure
	Parental involvement
	School/community support structure
Pedagogical Features	Academic achievement for all children
	Math and literature follow district linguistic policy
	Developmental level team teaching structures
	Thematic organization of units of study
	Teachers as monolingual models
	On-going staff development

Lindholm-Leary (2004) provided an excellent overview of two-way immersion programs. She asserts that two-way programs successfully educate native learners and English Language Learners within the same classroom and fulfill for both groups the goals of full bilingualism and biliteracy, grade-level academic achievement, and multicultural competency. Two-way immersion programs have four crucial features:

- Instruction and classwork take place in two languages, with the non-English language used for at least 50 percent of the students' instructional day.
- The day includes periods of instruction during which students and teachers use only one language, with no translation or language mixing allowed.
- Both English language learners and native English speakers do work in both languages in balanced proportion.
- English language learners and native English speakers are together for most content instruction.

Lindholm-Leary also presents six factors that influence the achievement of linguistically diverse students in bilingual programs:

- <u>School environment</u>: a cohesive, schoolwide vision with clearly defined goals for student achievement enhances student outcomes.
- <u>Curriculum and instruction</u>: any effective curriculum must be clearly aligned with standards and assessment and must be both meaningful and academically challenging.

- Program planning: a strong program-planning process should include proper scope, sequence, and alignment with developmentally appropriate practices and language proficiency levels in both languages.
- Assessment and accountability: Two Way Bilingual Instruction (TWBI) education programs should use multiple measures in both languages to assess students' progress toward bilingual and biliteracy goals, along with the curricular and content-related goals.
- Teacher quality and familiarity with bilingual education: Teachers must be familiar with the immersion model and with appropriate instructional strategies, and they must understand theories underlying bilingual education, second language development, cooperative learning, assessment, and education equity.
- Family involvement: Effective programs create an environment in which parents from all linguistic and cultural backgrounds feel valued and welcome.

Sheltered Instruction Observation Protocol

Short and Echevarria (1999) developed an explicit model of sheltered instruction that teachers could use to improve the academic success of their LEP students.

The protocol is defined as a tool derived from best practices identified in the research literature and on the experiences of participating teachers and researchers who collaborated in developing the observation protocol. The protocol is composed of 30 items grouped into three sections: Preparation, Instruction, and Review/Evaluation. Under Instruction, items are further clustered into the following subsections: Building Background, Comprehensible Input, Strategies, Interaction, Practice/Application, and Lesson Delivery. The protocol, which is used to provide feedback regarding classroom instructional behaviors, rates teachers using a Likert scale with scores ranging from 4 to 0 and uses descriptors of teaching behaviors to define the individual scoring options.

The Sheltered Instruction Observation Protocol (SIOP)

I. Preparation:

1. Write content objectives clearly for students
2. Write language objectives clearly for students
3. Choose content concepts appropriate for age and educational background level of students
4. Identify supplementary materials to use (graphs, models, visuals)
5. Adapt content (e.g., text, assignment) to all levels of student proficiency
6. Plan meaningful activities that integrate lesson concepts (e.g., surveys, letter writing, simulations, constructing models) with language practice opportunities for reading, writing, listening, and/or speaking

II. Instruction:

Building Background:

7. Explicitly link concepts to students' backgrounds and experiences
8. Explicitly link past learning and new concepts
9. Emphasize key vocabulary (e.g., introduce, write, repeat, and highlight) for students

Comprehensible Input:

10. Use speech appropriate for students' proficiency level (e.g., slower rate, enunciation, and simple sentence structure for beginners)
11. Explain academic tasks clearly
12. Use a variety of techniques to make content concepts clear (e.g., modeling, visuals, hands-on activities, demonstrations, gestures, body language)

Strategies:

13. Provide ample opportunities for students to use strategies (e.g., problem solving, predicting, organizing, summarizing, categorizing, evaluating, self-monitoring)
14. Use scaffolding techniques consistently (providing the right amount of support to move students from one level of understanding to higher level) throughout lesson
15. Use a variety of question types including those that promote higher-order thinking skills

throughout the lesson (e.g., literal, analytical, and interpretive questions)

Interaction:

16. Provide frequent opportunities for interaction and discussion between teacher/student and among students about lessons concepts, and encourage elaborated responses
17. Use group configurations that support language and content objectives of the lesson
18. Provide sufficient wait time for student responses consistently
19. Give ample opportunities for students to clarify key concepts in L1 as needed with aide, peer, or L1 text

Practice / Application:

20. Provide hands-on materials and/or manipulatives for students to practice using new content knowledge
21. Provide activities for students to apply content and language knowledge in the classroom
22. Provide activities that integrate all language skills (i.e., reading, writing, listening, and speaking)

Lesson Delivery:

23. Support content objectives clearly
24. Support language objectives clearly
25. Engage students approximately 90-100% of the period (most students taking part on task throughout the lesson)
26. Pace the lesson appropriately to the students' ability level.

III. Review / Evaluation:

27. Give a comprehensive review of key vocabulary
28. Give a comprehensive review of key content concepts
29. Provide feedback to students regularly on their output (e.g., language, content, work)
30. Conduct assessments of student comprehensions and learning throughout the lesson on all lesson objectives (e.g., spot checking, group response)

<u>Instructional Conversation</u>

In a report sponsored by the National Center for Research on Cultural Diversity and Second Language Learning, the topic of *instructional conversation* is presented, which Tharp and Gallimore (1991) believe is the key to "awakening and rousing to life the mental capacities of learners" (p. i).

The need for instructional conversations is based on the realization that "recitation" is the dominant instructional practice present in schools. This practice is described as

> Consisting of the teacher assigning a text (in the form of a textbook or a lecture) followed by a series of teacher questions that require students to display their mastery of the material through convergent factual answers. Recitation questioning seeks predictable, correct answers. It includes up to 20% "yes/no" questions. Only rarely in recitation are teacher questions responsive to student productions. Only rarely are they used to assist students to develop more complete or elaborated ideas, (p. 1).

According to these authors, recitation, as an instructional practice, needs to be replaced with teaching strategies that engage students in "conversation" or dialogue the critical form of assisting learners to develop thinking skills—the ability to form, express, and exchange ideas in speech and written form—though the questioning and sharing ideas and knowledge that happens in classroom conversations between teachers and students. Simply put, the authors state "to truly teach, one must converse; to truly converse is to teach."

Instructional conversations are based on the concept of learning contained in the work of Vygotsky. Vygotsky identified a "zone of proximal development" which is characterized as a zone of learning between what a student can do alone and what a student can do only with the assistance of another. Within this zone a student awakens a variety of internal development processes that are able to operate only when the child is interacting with people in his/her environment and in cooperation with his/her peers. It is, then, within the zone of proximal development that teaching may be defined as *assisted* *performance*—teaching occurs when a student's performance is achieved with assistance. The task of schools operating with this philosophy is to create and support instructional conversations among students and teachers in which teachers attempt to grasp the communicative intent of the child through intentional listening, to make guesses about the meaning of the intended communication (based on the context and on knowledge of the child's interests and experiences), and to adjust their responses to assist the child's efforts—in other words—to engage in conversation. When this occurs, classrooms are transformed into a community of learners, particularly when teachers reduce the distance between themselves and their students by constructing lessons from common understandings of each others' experience and ideas and make teaching a warm, interpersonal activity.

Goldenberg (1992) advocates the use of Instructional Conversations (ICs). He notes that instructional conversations are instructional in intent and are designed to promote learning. They are conversational in quality in that they appear to be natural and spontaneous language interactions. While engaging in instructional conversations teachers and students are responsive to what others say, so that each statement or contribution builds upon, challenges, or extends a previous one. Both teacher and students present provocative ideas or experiences, to which others respond.

Instructional conversations, according to Goldenberg, are in line with "constructivist" thinking. Therefore, students are expected to actively construct their own knowledge and understanding by making connections, building mental schemata, and developing new concepts from previous understandings. To this end, teachers encourage expression of students' own ideas, builds on information students provide, and generally guides students to increasingly sophisticated levels of comprehension.

Instructional conversations are heavily influenced on two ideas from the work of Vygotsky. First, instructional conversations should take place in the zone of proximal development, where children construct—with the assistance of a skilled teacher—understandings of important ideas, concepts, and texts they would not normally understand. Second, language is a primary vehicle for intellectual development. So that, language is not only a means for communicating information, it is also an important vehicle for helping learners broaden and deepen their understanding of important ideas.

To assist an understanding of instructional conversations, Goldenberg provides the following model, which includes two important parts: the instructional component and the conversational component.

- Instructional
    1. <u>Thematic Focus</u>: the teacher selects a theme on which to focus the discussion and has a general plan for how the theme will unfold.
    2. <u>Activation and Use of Background and Relevant Schemata</u>: the teacher either "hooks into" or provides students with pertinent background knowledge and schemata necessary for understanding.
    3. <u>Direct Teaching</u>: When necessary, the teacher provides direct teaching of a skill or a concept.
    4. <u>Promotion of More Complex Language and Expression</u>: the teacher elicits more extended student contributions by using a variety of elicitation techniques: invitations to expand, questions, restatements, and pauses.
    5. <u>Promotion of Bases for Statements or Positions</u>: the teacher promotes students' use of texts, pictures, and reasoning to support an argument or position, by gently probing.

- Conversational
    6. <u>Fewer Known-Answer Questions</u>: much of the discussion centers on questions for which there might be more than one correct answer.
    7. <u>Responsiveness to Student Contributions</u>: while having an initial plan and maintaining the focus and coherence of the discussion, the teacher is also responsive to students' statements and the opportunities they provide.
    8. <u>Connected Discourse</u>: the discussion is characterized by multiple, interactive, connected turns; succeeding utterances build upon and extend previous ones.

9. Challenging, but Non-threatening Atmosphere: the teacher creates a challenging atmosphere that is balanced by a positive affective climate. The teacher is more a collaborator than an evaluator, and students are challenged to negotiate and construct the meaning of the text, etc.
10. General Participation, Including Self-selected Turns: the teacher does not hold exclusive right to determine who talks; students are encouraged to volunteer or otherwise influence the selection of speaking turns.

In sum, instructional conversations could be thought of as a weaving guided by the teacher. First, a skilled teacher weaves together the comments and contributions made by different students with the ideas and concepts the teacher wishes to explore with them. Second, the teacher weaves students' prior knowledge and experiences with new knowledge and experiences, thereby broadening the scope of their understanding while building upon understandings they already possess. Finally, during the course of the conversation, the teacher weaves together, in appropriate proportions and shadings, the ten elements of the instructional conversations model.

Dalton and Sison (1995) conducted a study utilizing Instructional Conversations (ICs) to help increase the participation of seventh grade students who were ordinarily excluded from classroom participation by the teacher. By using ICs, the authors hoped to increase the participation in teaching and learning activities for teachers and language minority students though the active use of language and communication. The authors note that the challenge for teachers of language minority students striving to learn, simultaneously, both everyday conversation and the academic language of content areas in English, is to provide the experiences and significant amounts of time to develop language proficiency and academic achievement. However, language minority students may lack experience in classroom social interactions and are less likely to understand the rules (both implicit and explicit) of successful participation or strategies for achieving lesson goals. The result is that these students are excluded from interactive instruction and are thus unable to participate in the activity and language that build common understandings, self-esteem, and shared perspectives.

To overcome the challenges faced by low-achieving language minority students, the authors utilized instructional conversations, which is most often enacted in small groups using familiar forms of conversation to assist students' language production and understanding. Dalton and Sison visualize ICs as having the following characteristics or "faces":

Instructional Face	Conversational Face
• Outcomes-based	• Inclusive
• Assessment	• Responsive
• Assistance	• Joint Participation
Co-constructed Knowledge	

This study was conducted in a California middle school serving a population predominately of Mexican descent. Six seventh grade language minority students identified as low achieving participated in mathematics lessons in an English-only classroom. Students were characterized as being unteachable, having gang related histories, and erratic in attendance. Four math lessons were presented using ICs as the structure of the lesson, which were recorded and analyzed.

The results of the study indicate that the ratio of teacher talk and student talk shifted during the course of the study and that IC lessons encouraged students to talk. The results indicated an increase in students' comfort and skill in conversation; they were getting better with time. Students were using more content lexicon as the lessons progressed, which indicates that students' grasp of the concepts increased. Further, the language use strongly suggests that students' points of view shifted from those of an excluded outsider to those of the included member, the math student. Consequently, student interest and focus increased as well as mutual assistance and cooperation for learning purposes. The authors conclude that although the contact time for the teacher and students in these IC lessons was limited to four brief lessons interspersed over many weeks, there is evidence that the IC created new conditions for these students to participate in knowledge-constructing conversation.

Standards for professional development in the effective teaching of ELL students

This section is presented to anchor the literature of mathematics instruction. These standards are presented by the National Council of Teachers of Mathematics (NCTM) and the Center for Research on Education Diversity and Excellence (CREDE). The NCTM Standards are to be applied to all students and all instruction of mathematics. The CREDE standards were formulated with ELL students in mind. Similarities can be seen in both cases. When connected to the suggestions and strategies mentioned previously, the context for mathematics instruction becomes rich and challenging for all students, not just ELL students.

The *National Council of Teachers of Mathematics* (NCTM) standards established five goals for mathematical literacy:

1. that students learn to value mathematics,
2. that they became confident in their ability to do mathematics;
3. that they become mathematical problem solvers;
4. that they learn to communicate mathematically; and
5. that they learn to reason mathematically.

Additionally, the *NCTM Professional Standards for Teaching Mathematics* lays the groundwork for working with all students of mathematics:

1. Select mathematics tasks that engage students' interests and intellect.
2. Orchestra classroom discourse in ways that provide the investigation and growth of mathematical ideas.
3. Use, and help students use, technology and other tools to pursue mathematical investigations.
4. Seek, and help students seek, connections to previous and developing knowledge
5. Guide individual, small group, and whole class work.

The *Center for Research on Education Diversity and Excellence* has identified five standards for effective teaching of ELL students. These standards include: joint productive activity, language and literacy development, contextualizing teaching and learning, complex thinking, and instructional conversation. In a research brief Rueda (1998) discusses the five standards in terms of sociocultural theory and explains how each standard can support the learning process underlying professional development efforts.

1. <u>Facilitate learning and development through joint productive activity among leaders and participants</u>

   A sociocultural model for professional development therefore involves assisted performance by a more competent other. In this model the roles of student and teacher are more permeable and flexible than in models of professional development practice which rely on outside experts. Thus a one-shot workshop provided by an expert will not be as effective as a collaborative effort to solve a common problem.

   When thinking about professional development in terms of joint productive activity, joint refers to who is allowed to participate and how, while productive refers to what counts as a legitimate collaboration. It may help to make the rules for participation explicit

2. Promote learners' expertise in professionally relevant discourse

   In this view, thinking takes place through the medium of language, and language helps frame problems in new and important ways. Special discourse can be a central part of the professional development process, as long as it helps to frame a problem, capture a phenomenon with more precision, or reconceptualize it in a more useful or accessible way. Professional development should not involve jargon if it does not contribute to meaningful problem-solving, or if it has no connection to practice. Rather, professional development should work to create a common community of discourse. The leader needs to understand participants' discourse and vice versa.

3. Contextualize teaching, learning, and joint productive activity in the experiences and skills of participants

   Another premise of sociocultural theory is that teaching and learning must be contextualized, or situated in meaningful activities connected to everyday life. This means that teaching and learning activities and joint problem-solving tasks should focus on authentic issues and problems encountered in participants' daily practice. Both the problems addressed as well as the teaching and learning processes in these contexts are certain to be "messier" than those typically encountered in more controlled or artificial situations, but more meaningful to participants. Professional development should be flexible–to allow for local differences and diversity–and concrete–to avoid the syndrome of "that sounds good, but it won't work here." Innovations and school reform initiatives which rely upon rigid replication of a model or set of practices fail to account for the individual circumstances found in specific schools. They should be addressed through collaborative work.

4. Challenge participants toward more complex solutions in addressing problems

   There are many examples of teachers collaboratively addressing complex problems in innovative and successful ways (e.g., Clark, Hong, & Schoeppach, 1996). However, some school reform mandates have become more restrictive, constraining the ability of educational practitioners to develop locally meaningful solutions. Yet, the same high standards and meaningful feedback on efforts that are critical to students' success should be accorded to teachers. Professional development activities are better conceptualized as sustained problem-solving opportunities rather than short-term exercises designed to address simple issues. These are opportunities for members of professional development teams to seek responsive assistance as needed, either internally or from more competent others.

5. Engage participants through dialogue, especially the instructional conversation

   Instructional conversations are useful for creating responsive learning environments and should be utilized in professional development activities. The instructional conversation (IC) is a blend of deliberate, planned teaching with more interactive, responsive conversation. The instructional aspects of the IC are related to the opportunities for responsive assistance in the ongoing interactions among participants. The conversational aspects of the IC provide the hook that facilitates the connection of formal schooled knowledge to practical knowledge, including that which comes from teaching and being immersed in a community of teachers. In an important sense, this is at the heart of professional development: connecting the streams of classroom culture and knowledge with more formal knowledge and theory around collaborative problem-solving, that is, joint productive activity.

True teaching and learning take place only when these principles are in place, and professional development is a special case of teaching and learning. There may be a wide range of implementation options that may be faithful to these principles in ways that look very different from each other but may still be effective in the local ecology in which they develop. This should be seen as a natural outgrowth of the sociocultural emphasis on context as a major determinant of behavior.

Another formulation of the CREDE Standards comes from Tharp, Estrada, Dalton, and Yamauchi (2000). They propose the five *Standards for Effective Pedagogy* as critical for improving the learning outcomes of all students, and especially those of diverse ethnic, cultural, linguistic, or economic backgrounds. The five standards are:

1. Standard One: Teachers and Students Producing Together
    - *Facilitate learning through joint productive activity among teacher and students*
2. Standard Two: Developing Language and Literacy Across the Curriculum
    - *Develop competence in the language and literacy of instruction across the curriculum*
3. Standard Three: Making Meaning—Connecting School to Student's Lives
    - *Contextualize teaching and curriculum in the experiences and skills of students' homes and communities*
4. Standard Four: Teaching Complex Thinking
    - *Challenge students toward cognitive complexity*
5. Standard Five: Teaching Through Conversation
    - *Engage students through dialogue, especially the Instructional Conversation*

Issues Related to Mathematics Instruction

This section is included because it highlights an important problem in the instruction of mathematics—translation of word problems. This issue is present in the literature of mathematics and ELL students and is a point of argument for the importance of teaching mathematics within the context of literacy instruction; however, this is a universal problem in mathematics instruction for all grade levels of mathematics students. Given this fact, the question of mathematics instruction on the whole comes into question.

Clement, Lochhead, and Monk (1981) discuss a common problem among all students of mathematics—difficulties students face in translating stated (word) problems into mathematical notation (equations) so the problem can be solved. In a study with freshman engineering college students the authors found that fewer than 50 percent of the students could solve a series of written problems by translating the word problem into and out of algebraic notation. The authors believed that the challenge for students is not one of simply misunderstanding English. Rather, one source of the challenge is that secondary students are not getting exposure to such types of problems, but that teachers have tended to deemphasize such problems because students find them difficult. A second source is that students perform two types of errors in attempting to solve these problems: the first is word order matching, which is a literal, direct mapping of the words of English into the symbols of algebra; the second is the static-comparison method, which is a literal attempt to symbolize the static comparison between two groups. These challenges represent the students' reversal error in constructing the proper algebraic notation, to which the authors note that reversal difficulty appears to be rather resilient and requires considerable attention and discussion before students can learn to overcome it. They go on to admit that what makes teaching and learning of these translation skills so difficult is that behind them there are many unarticulated mental processes that guide one in constructing a new equation on paper.

In a similar study, Lochhead and Mestre (1988) examined the same challenge of students having difficulties in solving algebraic word problems, particularly in regard to translating written language into mathematical language. Citing the previous study, these authors note that the source of the error stems from misconceptions concerning the structure and interpretation of algebraic statements and of the process by which one translates between written language and algebraic language. The difficulty stems neither from a lack of algebraic fluency nor an inability to read. Rather, two types of errors are particularly of interest: first, students exhibit a strong proclivity toward performing a left-to-right word-order match when they translate the elements of a problem; second, students often confuse the distinction between variables and labels. Further, they lament that our educational system does not seem to address the conceptual issues that would help students overcome these types of misconceptions. In other words, students do not learn to read and write in mathematics.

Lochhead and Mestre offer several suggestions for overcoming these difficulties. First, teachers should provide students with ample practice at the translation process itself, isolated from all other aspects of problem solving. Since a certain amount of confusion is likely due to the different representational systems used in mathematics, full mastery of problem solving skills is possible only after students have struggled to make meanings among the confusion. Second, the authors propose a three step process for solving word problems that includes qualitative understanding, quantitative understanding, and conceptual understanding. In practice, say the authors, this approach resembles a Socratic dialogue, since the teacher seldom tells the student what the correct answer is but simply asks probing questions that attempt to elicit a contradiction resulting from the student's misconception. The student is then guided through additional probing questions, toward reaching a resolution to the contradiction. The goal is not necessarily to have the students write the appropriate equation but to have them grapple with, and dislodge, their misconceptions. These types of discussions are able to not only airing the different misconceptions students may have but also help students resolve their misconceptions through peer interactions.

Findings and Recommendations

The end result of this literature review is found in the findings and recommendations offered here. These are based on the documents presented in this literature review and are in keeping with the charge to identify what is known and what needs to be done in regard to the teaching of mathematics for ELL students.

Findings in the Literature:

- There is a paucity of research specific to mathematics and ELL students in general and for secondary ELL students in particular.
- In the current literature on mathematics and ELL students, the consensus is that mastery of content, the principles of literacy, and language acquisition are tied together—content, literacy, and language acquisition go hand-in-hand.
- Two-way dual language programs hold the most promise for delivery of all content to ELL students; one way language programs are the second most successful; all others fail to reach the 50% mark (that is, the mean score for all groups compared) and decline over time as well (see Thomas & Collier, 2002).
- Teachers (and teacher preparation programs) must acknowledge that mathematics has a formal content language register and the teaching of this register is critical to all students' understanding of math, beginning in the elementary school.
- The inability of all types of students to successfully translate word problems is a universal source of frustration for mathematics instruction in K-16, and is not just limited to ELL students; however, the language barrier of ELL students compounds their lack of understanding.
- Mathematics scores statewide are below acceptable levels in *all* student categories, particularly at Algebra I. This would seem to accentuate the existence an overall problem in mathematics instruction.

Findings Hinted at, but Not Discussed in the Literature:

- The compartmentalization of content, in this case mathematics, so that students fail to see connections across subject areas; this is particularly true in secondary education environments and in the case of ELL students.
- Teaching for understanding as a goal of instruction for all content areas. Teaching for understanding is inhibited by the previous point, but should be a recognized goal of instructional efforts at all grade levels.
- Teachers, particularly elementary teachers, may not be skilled in the technical aspects of mathematics *register* and how to teach it effectively and with consistency so that it provides the needed foundation for the teaching of secondary mathematics.
- The effects of power manifestation inherent in US school classrooms—English is dominant, all other languages are secondary; thus, traditional second language acquisition programs seek to change minority students to be like the majority. The exception to this reality is found in dual language programs, where native languages and English are seen as equally important, and all students benefit from instructional efforts in two languages.
- The role and influence of the principal in the viability and success of ELL programs.
- The role and influence of the teacher in the viability and success of ELL programs.

What is not Discussed in the Literature:

- The difference between how mathematics is taught in other countries and how it is taught in the US (i.e., conceptually-based vs. process-based; see the Third International Math & Science Studies for more support of this idea or *The Teaching Gap* by Hiebert & Stiggins).
- The acknowledgement that the poor math scores statewide in particular may be indicative of ineffective teaching strategies for all students and that the math curriculum needs to be revamped to reflect a different mode of instructional focus (i.e., cover less more deeply) and alternative instructional strategies that promote engagement, understanding, and higher order thinking.

Recommendations:

- Support and fund research to continue to identify programs that successfully tie literacy to content instruction, particularly as it concerns mathematics, such as SLAMS (Cuevas, 1981, 1984, 1992).
- Support and fund the development of Two-way dual language programs at all school levels, not just elementary schools where these programs are typically found.
- Support and fund programs that build on the positives of language acquisition for all students at all grade levels rather than programs that isolate students because of their native language and seek to change them to English speakers primarily.
- Support and fund programs to identify and implement instructional strategies that engage all students such as Instructional Conversations, Concept/thematic instructional practices, etc.
- Support and fund professional development for teachers regarding needs of ELL students and instructional practices that support ELL students (see CREDE Five Standards for Effective Pedagogy and Student Outcomes).
- Examine teacher preparation programs; require elementary teachers to have formal training in math instruction, particularly in learning the mathematics register, and/or in instructional strategies that facilitate development of literacy and mathematics skills.
- Recognize and support the importance of mathematics pedagogy in teacher preparation courses and programs. Demand for teacher mastery of mathematics is seen as a crucial to being able to teach mathematics register, concepts, and processes. However, an equal assertion can be made that there is a difference in *learning* mathematics and *teaching* mathematics. Teachers must not only understand mathematics, they must be able to *teach* in a way that students can understand mathematics. That is, teachers must receive training in mathematics pedagogy.
- Support changes in math instruction from process to conceptual foundations for all K-12 students.
- Rewrite math TEKS to support *depth* rather than *breadth* in instructional focus. Further, support the conceptual understanding of mathematics through revamping of TAKS testing of mathematics.

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